Digital Signal Processing
Introduction

1. Course overview
2. Digital Signal Processing
3. Basic operations & block diagrams
4. Classes of sequences
1. Course overview

- Digital signal processing: Modifying signals with computers
- Book: Oppenheim “Discrete time Signal Processing”
- Instructor: manzuri@sharif.edu
2. Signals

- Signals:
  Information-bearing function

- E.g. sound: air pressure variation at a point as a function of time $p(t)$

- Dimensionality:
  Sound: 1-Dimension
  Greyscale image $i(x,y) : 2$-D
  Video: $3 \times 3$-D: $\{r(x,y,t) \ g(x,y,t) \ b(x,y,t)\}$
3. Examples

- Noise - all domains
- Spread-spectrum phone - radio
- ECG - biological
- Music
- Image/video - compression
- ....
4. Signal processing

- Modify a signal to extract/enhance/rearrange the information
- Origin in analog electronics e.g. radar
- Examples...
  - Noise reduction
  - Data compression
  - Representation for recognition/classification...
5. DSP vs. analog SP

- Conventional signal processing:

\[ p(t) \rightarrow \text{Processor} \rightarrow q(t) \]

- Digital SP system:

\[ p(t) \rightarrow A/D \rightarrow p[n] \rightarrow \text{Processor} \rightarrow q[n] \rightarrow D/A \rightarrow q(t) \]
6. DSP example

- Speech time-scale modification: extend duration without altering pitch
7. Operations on signals

- Discrete time signal often obtained by sampling a continuous-time signal

- Sequence \( \{x[n]\} = x_d(nT), n=\ldots-1,0,1,2\ldots \)

- \( T = \) samp. period; \( 1/T = \) samp. frequency
Sequences

- Can write a sequence by listing values:
  \[ \{x[n]\} = \{\ldots, -0.2, 2.2, 1.1, 0.2, -3.7, 2.9, \ldots\} \]
  \[ \uparrow \]
- Arrow indicates where \( n=0 \)
- Thus, \( x[-1] = -0.2, \ x[0] = 2.2, \ x[1] = 1.1, \)
Operations on sequences

- **Addition** operation:
  
  ![Addition Diagram]
  
  $y[n] = x[n] + w[n]$

- **Adder**

- **Multiplication** operation

  ![Multiplication Diagram]
  
  $y[n] = A \cdot x[n]$

- **Multiplier**
More operations

- **Product (modulation) operation:**

  ![Diagram](image)

  \[ y[n] = x[n] \cdot w[n] \]

- Modulator

- E.g. **Windowing**: multiplying an infinite-length sequence by a finite-length window sequence to extract a region.
Time shifting

- **Time-shifting** operation: \( y[n] = x[n - N] \)
  where \( N \) is an integer

- If \( N > 0 \), it is **delaying** operation
  - Unit delay: \( x[n] \xrightarrow{\mathcal{Z}^{-1}} y[n] \)
    \[ y[n] = x[n - 1] \]

- If \( N < 0 \), it is an **advance** operation
  - Unit advance: \( x[n] \xrightarrow{\mathcal{Z}} y[n] \)
    \[ y[n] = x[n + 1] \]
Combination of basic operations

- Example

\[ y[n] = \alpha_1 x[n] + \alpha_2 x[n - 1] + \alpha_3 x[n - 2] + \alpha_4 x[n - 3] \]
Up- and down-sampling

- Certain operations change the effective sampling rate of sequences by adding or removing samples
- Up-sampling = adding more samples = interpolation
- Down-sampling = discarding samples = decimation
Down-sampling

- In **down-sampling** by an integer factor $M > 1$, every $M$-th samples of the input sequence are kept and $M - 1$ in-between samples are removed:

$$y[n] = x[nM]$$

$$x[n] \quad \downarrow M \quad y[n]$$
Down-sampling

- An example of down-sampling
Up-sampling

Up-sampling is the converse of down-sampling: $L$-1 zero values are inserted between each pair of original values.

$$x_u[n] = \begin{cases} x[n/L], & n = 0, \pm L, \pm 2L, \ldots \\ 0, & \text{otherwise} \end{cases}$$
Up-sampling

- An example of up-sampling
8. Complex Numbers

- Complex numbers are a mathematical convenience that lead to simple expressions. A 2nd “imaginary” dimension ($j \equiv \sqrt{-1}$) is added to all values.

- **Rectangular form:** $x = x_{re} + j \cdot x_{im}$
  - where *magnitude* $|x| = \sqrt{(x_{re}^2 + x_{im}^2)}$
  - and *phase* $\theta = \tan^{-1}(x_{im}/x_{re})$

- **Polar form:** $x = |x| e^{j\theta} = |x|\cos\theta + j \cdot |x|\sin\theta$
Complex math

- When **adding**, real and imaginary parts add: \((a+jb) + (c+jd) = (a+c) + j(b+d)\)

- When **multiplying**, magnitudes multiply and phases add: \(re^{j\theta} \cdot se^{j\phi} = rse^{j(\theta+\phi)}\)

- Phases modulo \(2\pi\)
Complex conjugate

- Flips imaginary part / negates phase: \( x^* = x_{re} - j \cdot x_{im} = |x| \cdot e^{j(-\theta)} \)

- Useful in resolving to real quantities:
  \[
  x + x^* = x_{re} + j \cdot x_{im} + x_{re} - j \cdot x_{im} = 2x_{re}
  \]
  \[
  x \cdot x^* = |x| \cdot e^{j(\theta)} \cdot |x| \cdot e^{j(-\theta)} = |x|^2
  \]
Classes of sequences

- Useful to define broad categories...
- Finite/infinite (extent in $n$)
- Real/complex:
  \[ x[n] = x_{re}[n] + j \cdot x_{im}[n] \]
Classification by symmetry

- Conjugate symmetric sequence:
  \[ x[n] = x^*[-n] = x_{re}[-n] - j \cdot x_{im}[-n] \]

- Conjugate antisymmetric:
  \[ x[n] = -x^*[-n] = -x_{re}[-n] + j \cdot x_{im}[-n] \]
Conjugate symmetric decomposition

- Any sequence can be expressed as conjugate symmetric (CS) / antisymmetric (CA) parts:
  \[ x[n] = x_{cs}[n] + x_{ca}[n] \]
  where:
  \[ x_{cs}[n] = \frac{1}{2}(x[n] + x^*[-n]) = x_{cs}^*[-n] \]
  \[ x_{ca}[n] = \frac{1}{2}(x[n] - x^*[-n]) = -x_{ca}^*[-n] \]

- When signals are real, CS => Even \((x_{re}[n] = x_{re}[-n])\), CA => Odd
Basic sequences

- **Unit sample sequence** - \( \delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases} \)

- **Shift in time**: \( \delta[n - k] \)

- **Can express any sequence with** \( \delta \): 
  \[ \{\alpha_0, \alpha_1, \alpha_2, \ldots \} = \alpha_0 \delta[n] + \alpha_1 \delta[n-1] + \alpha_2 \delta[n-2] \ldots \]
More basic sequences

- Unit step sequence - $\mu[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$

- Related to unit sample:
  
  $\delta[n] = \mu[n] - \mu[n-1]$  
  $\mu[n] = \sum_{k=-\infty}^{n} \delta[k]$
Exponential sequences

- Exponential sequences = eigenfunctions
- General form: $x[n] = A \cdot \alpha^n$
- If $A$ and $\alpha$ are real:
Complex exponentials

\[ x[n] = A \cdot \alpha^n \]

- Constants \( A, \alpha \) can be complex

\[ A = |A|e^{j\phi}; \quad \alpha = e^{(\sigma + j\omega)} \]

\[ \Rightarrow x[n] = |A|e^{\sigma n}e^{j(\omega n + \phi)} \]

scale, varying magnitude, varying phase
Complex exponentials

- Complex exponential sequence can ‘project down’ onto real & imag axes to give sinusoidal sequences

\[ x[n] = \exp\left(-\frac{1}{12} + j\frac{\pi}{6}\right)n \]
Periodic sequences

A sequence $\bar{x}[n]$ satisfying $\bar{x}[n] = \bar{x}[n + kN]$, is called a periodic sequence with a period $N$ where $N$ is a positive integer and $k$ is any integer. Smallest value of $N$ satisfying $\bar{x}[n] = \bar{x}[n + kN]$ is called the fundamental period.
Periodic exponentials

- Sinusoidal sequence $A \cos(\omega_o n + \phi)$ and complex exponential sequence $B \exp(j \omega_o n)$ are periodic sequences of period $N$ only if $\omega_o N = 2\pi r$, with $N$ & $r$ positive integers.

- Smallest value of $N$ satisfying $\omega_o N = 2\pi r$ is the **fundamental period** of the sequence.

- $r = 1 \Rightarrow$ one sinusoid cycle per $N$ samples
- $r > 1 \Rightarrow r$ cycles per $N$ samples
Aliasing

- E.g. for $\cos(\omega n)$, $\omega = 2\pi r \pm \omega_0$ all appear the same after sampling
- We say that a larger $\omega$ appears as aliased to a lower frequency