پاسخ تمرین فصل پنجم سیگنال‌ها و سیستم‌ها :

1) $x_1[n] = \left(\frac{1}{2}\right)^{|n|} u[-n-2]$ (a

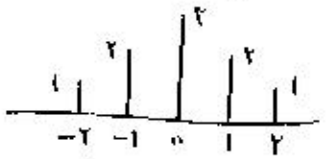
$$X_1(e^{j\omega}) = \frac{1}{2}\left[\frac{e^{j2\omega}}{1-\frac{1}{3}e^{j\omega}}\right]$$

$x_2[n] = \left(\frac{1}{2}\right)^{|n|} \cos\left(\frac{\pi}{8}(n-1)\right)$ (b

$$X_2(e^{j\omega}) = \frac{\left(\frac{1}{4}\right)e^{j(\omega-\frac{\pi}{4})}}{1-\frac{1}{2}e^{j(\omega-\frac{\pi}{8})}} + \frac{\left(\frac{1}{4}\right)e^{j(\omega+\frac{\pi}{4})}}{1-\frac{1}{2}e^{j(\omega+\frac{\pi}{8})}}$$

$$+\frac{\left(\frac{1}{2}\right)e^{-j\frac{\pi}{8}}}{1-\frac{1}{2}e^{-j(\omega-\frac{\pi}{8})}} + \frac{\left(\frac{1}{2}\right)e^{j\frac{\pi}{8}}}{1-\frac{1}{2}e^{-j(\omega+\frac{\pi}{8})}}$$

$x_3[n] =$ (c



$$X_3(e^{j\omega}) = e^{2j\omega} + 2e^{j\omega} + 3 + 2e^{-j\omega} + e^{-2j\omega}$$

(d

$$x_4[n] = \cos\left(\frac{\pi}{8}n\right)$$

$$X_4(e^{j\omega}) = \pi \sum_{k=-\infty}^{+\infty} \left(\delta(\omega-\frac{\pi}{8}-2k\pi) + \delta(\omega+\frac{\pi}{8}-2k\pi)\right)$$

(e

$$x_5[n] = \left(\delta[n-1] + \delta[n+1]\right)^{10}$$

$$X_5(e^{j\omega}) = e^{-j\omega} + e^{j\omega} = \cos(j\omega)$$

2) $$X(e^{j0}) = \sum_{n=-\infty}^{+\infty} x[n] = 6$$ (a

$y[n] = x[n+2] \Rightarrow y[n]$ even & real (b

$$\Rightarrow \angle Y(e^{j\omega}) = 0 \Rightarrow X(e^{j\omega}) = e^{2j\omega} \times Y(e^{j\omega})$$

$$\Rightarrow \angle X(e^{j\omega}) = \angle e^{2j\omega} + \angle Y(e^{j\omega}) = 2\omega + 0 \Rightarrow \angle X(e^{j\omega}) = 2\omega$$

(c

$$\int_{-\pi}^{\pi} X(e^{j\omega})\,d\omega = 2\pi x[0] = 4\pi$$

(d $X(e^{j\pi}) = \sum_{n=-\infty}^{+\infty} x[n](-1)^n = 2$

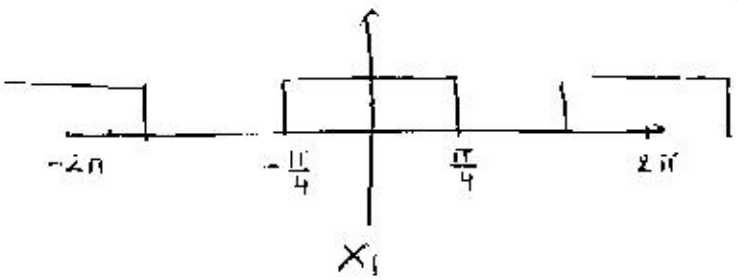
(e $Ev\{x[n]\}$

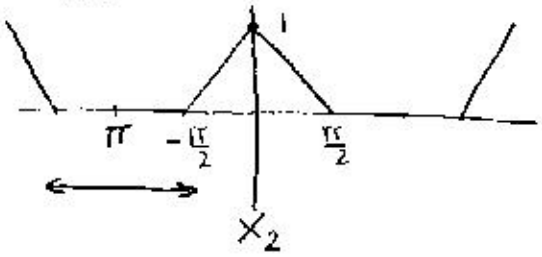
(f i) $\int_{-\pi}^{\pi} |X(e^{jw})|^2 dw = 2\pi \sum_{n=-\infty}^{+\infty} |x[n]|^2 = 28\pi$

ii) $\int_{-\pi}^{\pi} \left| \frac{dX(e^{jw})}{dw} \right|^2 dw = 2\pi \sum_{n=-\infty}^{+\infty} |-jnx[n]|^2 = 316\pi$

3) $X_1(e^{jw}) = F\left\{ \frac{\sin\frac{\pi}{4}n}{\pi n} \right\} = \begin{cases} 1 & |w| \le \frac{\pi}{4} \\ 0 & \frac{\pi}{4} < w < \pi \end{cases}$

$X_2(e^{jw}) = F\left\{ \left(\frac{\sin\frac{\pi}{4}n}{\pi n} \right)^2 \right\} = \frac{1}{2\pi} \int_{-\pi}^{\pi} X_1(e^{jw}) * X_1(e^{jw}) dw$

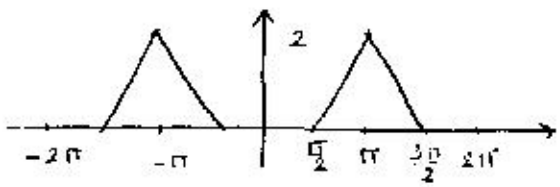




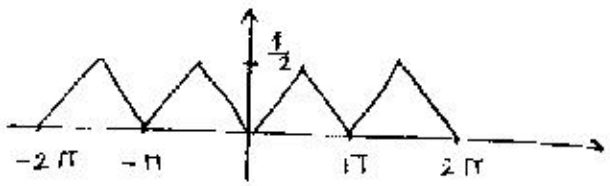
با اعاب $|w_c| \ge \frac{\pi}{2}$ ، رابطه‌ی مورد نظر برقرار خواهد بود .

4) (a

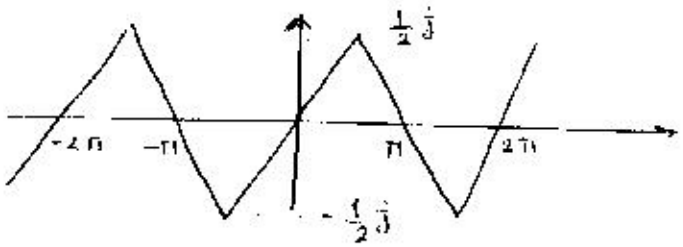
(i



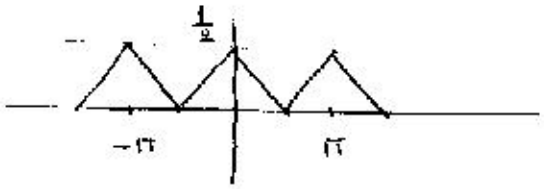
(ii



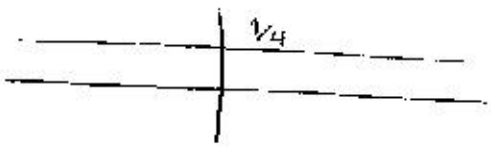
(iii



(iv)

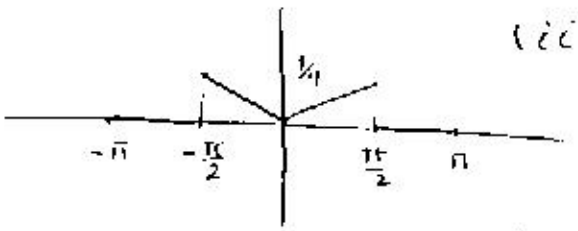


(v)



$$y[n] = e^{-j\frac{\pi}{4}n}\frac{\sin\frac{\pi n}{8}}{n^2} - e^{j\frac{\pi}{4}n}\frac{\sin\frac{\pi n}{8}}{n^2}$$

(iii)

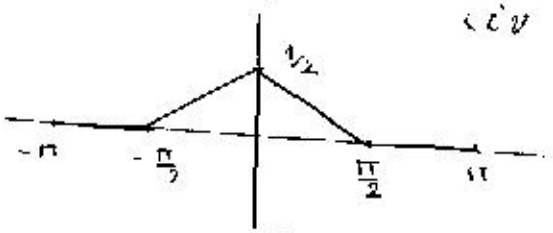


(b)

(i)

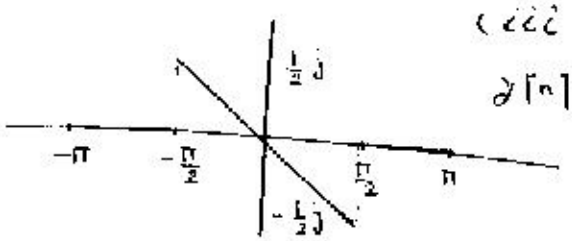
$y[n] = n$

(iv)



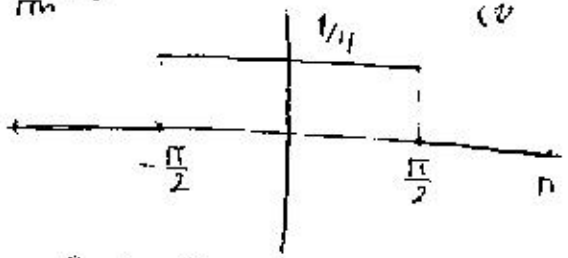
$$y[n] = \frac{1}{2}\left(\frac{\sin\frac{\pi}{4}n}{\pi n}\right)^2$$

(iii)



$$y[n] = \frac{\sin\frac{\pi}{2}n}{n^2}$$

(v)



$$y[n] = \frac{1}{4}\sin\left(\frac{\pi}{2}n\right)/\pi n$$

5) $$y_1[n] = \begin{cases} x\left[\frac{n}{M}\right] & n = kM \\ 0 & \text{otherwise} \end{cases}$$

$$Y(e^{j\omega}) = \sum_{m=-\infty}^{\infty} y[m]e^{-j\omega m} = \sum_{\substack{m=-\infty \\ m=nM}}^{+\infty} y[m]e^{-j\omega m} = \sum_{\substack{m=-\infty \\ m=nM}}^{+\infty} x\left[\frac{m}{M}\right]e^{-j\omega m}$$

$$= \sum_{n=-\infty}^{+\infty} x\left[\frac{nM}{M}\right]e^{-j\omega Mn} = X(e^{jM\omega})$$