Assignment #7 solutions:

1.

$$\omega_x \succ 2\omega_m \rightarrow \omega_s \succ 2(\omega_1+\omega_2) \rightarrow T \le \frac{\pi}{\omega_1+\omega_2}$$

2.

$$X(j\omega)=2\pi\sum a_k\delta(\omega-20k\pi) \rightarrow X_c(j\omega)=2\pi\sum_{-10}^{10} a_k\delta(\omega-20k\pi) \rightarrow x_c(t)=\sum_{-10}^{10} a_k e^{j20k\pi t}$$

$$\rightarrow x[n]=x_c(nT)=\sum_{-10}^{10} a_k e^{j20k\pi nT} \rightarrow b_k=\begin{cases} a_k & |k|\prec 10 \\ \frac{1}{2}(a_{10}+a_{-10}) & k=10 \end{cases}$$

3.

$$x_c(t)=u(t) \rightarrow X_c(j\omega)=\frac{1}{j\omega}+\pi\delta(\omega) \rightarrow Y_c(j\omega)=X_c(j\omega).H_c(j\omega)=\frac{1}{j\omega}+\pi\delta(\omega) \quad |\omega|\prec\frac{\pi}{4T}$$

$$\rightarrow y_c(t)=\wp^{-1}(Y_c(j\omega))$$

4.

$$a) Y(j\omega)=X_1(j\omega).X_2(j\omega) \rightarrow \omega \succ \min(\omega_1,\omega_2) \rightarrow \omega_s \succ 2\min(\omega_1,\omega_2)$$

$$b) Y(j\omega)=X_1(j\omega)-X_2(j\omega) \rightarrow \omega \succ \max(\omega_1,\omega_2) \rightarrow \omega_s \succ 2\max(\omega_1,\omega_2)$$

5.

$$Y(j\omega)=\sum X(j\omega)\delta(\omega-k\omega_0)=\sum X(jk\omega_0)\delta(\omega-k\omega_0)=\frac{1}{\omega_0}X(j\omega)P(j\omega)$$

$$\rightarrow y(t)=\frac{1}{\omega_0}x(t)*\wp^{-1}(P(j\omega))=\frac{1}{\omega_0}\sum x(kT)\delta(t-KT)$$

$$\rightarrow 2\omega_M \le \omega_0 \rightarrow \omega_M \le \frac{\omega_0}{2}$$

6.

$$x_2[n]=x[2n] \rightarrow X_2(e^{j\omega})=\frac{1}{2}(X(e^{j\frac{\omega}{2}})+X(e^{j\frac{\omega-2\pi}{M}}))$$

$$x_3[n]=x_2\left[\frac{n}{3}\right] \rightarrow X_3(e^{j\omega})=X_2(e^{j3\omega})=\frac{1}{2}(X(e^{j\frac{3\omega}{2}})+X(e^{j\frac{3\omega-2\pi}{M}}))$$

7.

$$\begin{bmatrix} x[n] \rightarrow x[nM] \rightarrow x\left[\frac{nM}{L}\right] \rightarrow nM=kL \\ x[n] \rightarrow x\left[\frac{n}{L}\right] \rightarrow x\left[\frac{nM}{L}\right] \rightarrow n=k'L \end{bmatrix} \rightarrow (nM=kL \Leftrightarrow n=k'L) \rightarrow \gcd(L,M)=1$$