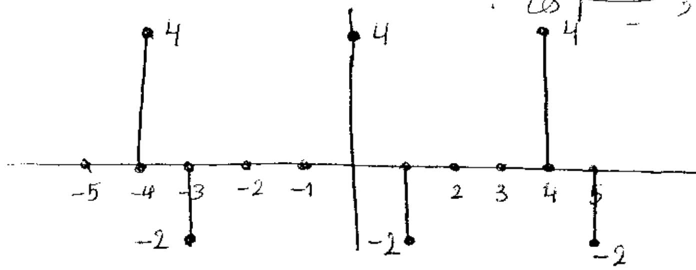
پاسخ سوالات پایان ترم سیگنال‌ها و سیستم‌ها :

۱- a)




b)

$$\sum_{n=13}^{15} v[n] = -2$$

$$v[n] = x[n] * h_1[n] = \left(\sum_{m=-\infty}^{+\infty} 2\delta[n-2-4m]\right) * (2\delta[n-2] - \delta[n-3])$$

$$= \sum_{m=-\infty}^{+\infty} 2\delta[n-2-4m] * (2\delta[n-2] - \delta[n-3])$$

$$= \sum_{m=-\infty}^{+\infty} 4\delta[n-4-4m] - 2\delta[n-5-4m] = \sum_{m=-\infty}^{+\infty} 4\delta[n-4m] - 2\delta[n-1-4m$$

$$v[13] = -2 \text{ , } v[14] = v[15] = 0 \implies \sum_{n=13}^{15} v[n] = -2$$

c)

$$y[n] = x[n] * h_1[n] * h_2[n]$$

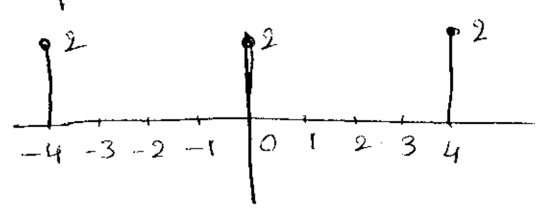
$$h_1[n] * h_2[n] = (2\delta[n-2] - \delta[n-3]) * \left(\tfrac{1}{2}\right)^{n+1} u[n]$$

$$= 2\left(\tfrac{1}{2}\right)^{n-2+1} u[n-2] - \left(\tfrac{1}{2}\right)^{n-3+1} u[n-3] = \left(\tfrac{1}{2}\right)^{n-2} u[n-2] - \left(\tfrac{1}{2}\right)^{n-2} u[n$$

$$= \left(\tfrac{1}{2}\right)^{n-2} (u[n-2] - u[n-3]) = \left(\tfrac{1}{2}\right)^{n-2} \delta[n-2] = \delta[n-2]$$

$$\implies y[n] = \delta[n-2] * \sum_{m=-\infty}^{+\infty} 2\delta[n-2-4m] =$$

$$\sum_{m=-\infty}^{+\infty} 2\delta[n-4-4m] = \sum_{m=-\infty}^{+\infty} 2\delta[n-4m]$$




۲-

System A : $y(t) = x(t+2)\sin(\omega t+2)$ , where $\omega \neq 0$

System B : $y[n] = \left(-\tfrac{1}{2}\right)^n (x[n]+1)$

a) خطی بودن :

System A : ✓

$$x_1(t) \longrightarrow y_1(t) = x_1(t+2)\sin(\omega t+2)$$

$$x_2(t) \longrightarrow y_2(t) = x_2(t+2)\sin(\omega t+2)$$

$$x_3(t) = a x_1(t) + b x_2(t) \longrightarrow y_3(t) = x_3(t+2)\sin(\omega t+2)$$

$$= a x_1(t+2)\sin(\omega t+2) + b x_2(t+2)\sin(\omega t+2) = a y_1(t) + b y_2(t)$$

System B : ✗ if $x[n] = 0$ then $y[n] \neq 0$

b) مستقل از زمان بودن :

System A : ✗ چون ورودی سیستم با زمان تغییر می‌کند .

System B : ✗ //

c) علی بودن :

System A : ✗ $y(t)$ depends on $x(t+2)$

System B : ✓ $y[n]$ depends only on the current value of $x[n]$

d) پایداری

System A : ✓ $|x(t)| < B \Longrightarrow |y(t)| < B$ ✓

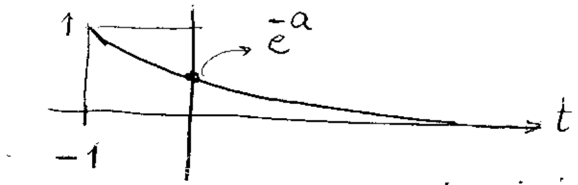
System B : ✗ $(-\frac{1}{2})^n \longrightarrow \infty$ as $n \longrightarrow -\infty$

۳- a) $\left(\frac{dy(t)}{dt} = -ay(t) + x(t+1)\right)$

حدس : $h(t) = e^{-a(t+1)} u(t+1)$

$$\frac{dh(t)}{dt} = -a e^{-a(t+1)} u(t+1) + e^{-a(t+1)} \delta(t+1) = -a h(t) + \delta(t+1)$$

b-




c- علی نیست چون برای $h(-\frac{1}{2})$ مخالف صفر است .

d-

$$\int_{-\infty}^{\infty} |h(t)|\, dt < \infty \Longleftrightarrow \text{BIBO stable}$$

$$\int_{-\infty}^{\infty} |h(t)|\, dt = \int_{-1}^{\infty} e^{-a(t+1)} dt = \int_{0}^{\infty} e^{-at} dt = -\frac{1}{a} e^{-at} \Big|_{0}^{\infty}$$

$$= -\frac{1}{a}(0-1) \quad \text{if } a>0$$

$$\begin{cases} \frac{1}{a} & \text{if } a>0 \\ \infty & \text{if } a \le 0 \end{cases}$$

$$\boxed{a>0 \Longleftrightarrow \text{BIBO stable}}$$