TABLE 3.1 PROPERTIES OF CONTINUOUS-TIME FOURIER SERIES		
PROPERTY	SIGNAL $x(t), y(t)$: *T-periodic*, $\omega_0 = 2\pi / T$	FOURIER SERIES COEFFICIENTS a_k, b_k
Linearity	$Ax(t)+By(t)$	Aa_k+Bb_k
Time-Shifting	$x(t-t_0)$	$a_k e^{-jk\omega_0 t_0}$
Frequency-Shifting	$e^{jM\omega_0 t} x(t)$	a_{k-M}
Conjugation	$x^*(t)$	a^*_{-k}
Time-Reversal	$x(-t)$	a_{-k}
Time-Scaling	$x(at)$, $a>0$ (T/a-periodic)	a_k
Convolution	$h(t)*x(t)$ $\mathbb{F}\{h\}=H(j\omega)$	$H(jk\omega_0)a_k$
Periodic convolution	$\int_T x(\tau) y(t-\tau) d\tau$	$Ta_k b_k$
Multiplication	$x(t)y(t)$	$\sum_{l=-\infty}^{\infty} a_l b_{k-l}$
Differentiation	$dx(t)/dt$	$jkw_0 a_k$
Integration	$\int_{-\infty}^{t} x(\tau) d\tau$ (finite valued and periodic only if $a_0 = 0$)	$a_k / jk\omega_0$
Conjugate Symmetry	$x(t)$ real	$a_k = a^*_{-k}$, $Re\{a_k\}$ even; $Im\{a_k\}$ odd $\lvert a_k \rvert$ even; $arg(a_k)$ odd
	$x(t)$ real and even	a_k real and even
	$x(t)$ real and odd	a_k pure-imag. and odd
	$Even\{x(t)\}$ ($x(t)$ real)	$Re\{a_k\}$
	$Odd\{x(t)\}$ ($x(t)$ real)	$jIm\{a_k\}$
Parseval	$\frac{1}{T}\int_T \lvert x(t) \rvert^2 dt = \sum_{k=-\infty}^{\infty} \lvert a_k \rvert^2$	

TABLE 3.2 PROPERTIES OF DISCRETE-TIME FOURIER SERIES

Property	Periodic Signal	Fourier Series Coefficients
	$\left.\begin{matrix} x[n] \\ y[n] \end{matrix}\right\}$ Periodic with period N and fundamental frequency $\omega_0 = 2\pi/N$	$\left.\begin{matrix} a_k \\ b_k \end{matrix}\right\}$ Periodic with period N
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time Shifting	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shifting	$e^{jM(2\pi/N)n} x[n]$	a_{k-M}
Conjugation	$x^*[n]$	a^*_{-k}
Time Reversal	$x[-n]$	a_{-k}
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m], & \text{if } n \text{ is a multiple of } m \\ 0, & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period mN)	$\frac{1}{m} a_k \begin{pmatrix}\text{viewed as periodic} \\ \text{with period } mN\end{pmatrix}$
Periodic Convolution	$\sum_{r=\langle N\rangle} x[r]y[n-r]$	$Na_k b_k$
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N\rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n-1]$	$(1 - e^{-jk(2\pi/N)})a_k$
Running Sum	$\sum_{k=-\infty}^{n} x[k] \begin{pmatrix}\text{finite valued and periodic only} \\ \text{if } a_0 = 0\end{pmatrix}$	$\left(\frac{1}{(1 - e^{-jk(2\pi/N)})}\right)a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a^*_{-k} \\ \Re e\{a_k\} = \Re e\{a_{-k}\} \\ \Im m\{a_k\} = -\Im m\{a_{-k}\} \\ \lvert a_k\rvert = \lvert a_{-k}\rvert \\ \sphericalangle a_k = -\sphericalangle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	a_k real and even
Real and Odd Signals	$x[n]$ real and odd	a_k purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e[n] = \mathcal{E}v\{x[n]\} & [x[n] \text{ real}] \\ x_o[n] = \mathcal{O}d\{x[n]\} & [x[n] \text{ real}] \end{cases}$	$\Re e\{a_k\}$ $j\Im m\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N}\sum_{n=\langle N\rangle} |x[n]|^2 = \sum_{k=\langle N\rangle} |a_k|^2$$

TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM		
$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}d\omega$		$X(j\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}dt$
PROPERTY	APERIODIC SIGNAL $x(t)$, $y(t)$	FOURIER TRANSFORM $X(j\omega)$, $Y(j\omega)$
Linearity	$ax(t)+by(t)$	$aX(j\omega)+bY(j\omega)$
Time-Shifting	$x(t-t_0)$	$X(j\omega)e^{-j\omega t_0}$
Frequency-Shifting	$e^{j\omega_0 t}x(t)$	$X(j(\omega-\omega_0))$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time-Scaling	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{j\omega}{a}\right)$
Convolution	$x(t)*y(t)$	$X(j\omega)Y(j\omega)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}X(j\omega)*Y(j\omega)$
Differentiation	$dx(t)/dt$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^{t}x(\tau)d\tau$	$\frac{1}{j\omega}X(j\omega)+\pi X(0)\delta(\omega)$
Differentiation in frequency	$tx(t)$	$j\frac{dX(j\omega)}{d\omega}$
Conjugate Symmetry	$x(t)$ real	$X(j\omega)=X^*(-j\omega)$, $Re\{X(j\omega)\}$ even; $Im\{X(j\omega)\}$ odd $\|X(j\omega)\|$ even; $arg(X(j\omega))$ odd
	$x(t)$ real and even	$X(j\omega)$ real and even
	$x(t)$ real and odd	$X(j\omega)$ pure-imag. and odd
	$Even\{x(t)\}$ ($x(t)$ real)	$Re\{X(j\omega)\}$
	$Odd\{x(t)\}$ ($x(t)$ real)	$jIm\{X(j\omega)\}$
Parseval	$\int_{-\infty}^{\infty}\|x(t)\|^2\,dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\|X(j\omega)\|^2\,d\omega$	

TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS		
SIGNAL	FOURIER TRANSFORM	FOURIER SERIES COEFF. (if periodic)
$\sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$	$2\pi\sum_{k=-\infty}^{\infty} a_k \delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$a_1 = 1$, $a_k = 0$ otherwise
$\cos \omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$	$a_1 = a_{-1} = ½$ $a_k = 0$ otherwise
$\sin \omega_0 t$	$(\pi/j)[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$	$a_1 = -a_{-1} = 1/(2j)$ $a_k = 0$ otherwise
$x(t) = 1$	$2\pi\delta(\omega)$	$a_0 = 1$, $a_k = 0$ otherwise
Periodic square wave $x(t) = \begin{cases} 1, & \|t\| < T_1 \\ 0, & T/2 > \|t\| > T_1 \end{cases}$ $x(t+T) = x(t)$	$\sum_{k=-\infty}^{\infty} \frac{2\sin k\omega_0 T_1}{k}\delta(\omega - k\omega_0)$	$\frac{\sin k\omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\frac{2\pi}{T}\sum_{k=-\infty}^{\infty}\delta\left(\omega - \frac{2\pi k}{T}\right)$	$a_k = 1/T$, for all k
$x(t) = \begin{cases} 1, & \|t\| < T_1 \\ 0, & \|t\| > T_1 \end{cases}$	$\frac{2\sin \omega T_1}{\omega}$	-
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \|\omega\| < W \\ 0, & \|\omega\| > W \end{cases}$	-
$\delta(t)$	1	-
$U(t)$	$1/(j\omega) + \pi\delta(\omega)$	-
$\delta(t-t_0)$	$e^{-j\omega t_0}$	-
$e^{-at}U(t)$, $Re\{a\}>0$	$1/(a+j\omega)$	-
$te^{-at}U(t)$, $Re\{a\}>0$	$1/(a+j\omega)^2$	-
$\frac{t^{n-1}}{(n-1)!}e^{-at}U(t)$, $Re\{a\}>0$	$1/(a+j\omega)^n$	-