1. For a multiclass problem, if any class $S_k$ can be separated from all other classes $S_j$ by a single hyper plane, the samples are totally linearly separable and can be separated by $S_k$ vs. not $S_k$ decisions. Let the decision functions for a 3 class problem be 
\[g_1(x) = -x_1 + x_2\]
\[g_2(x) = x_1 + x_2 - 5\]
\[g_3(x) = -x_2 + 1\]

Let
\[g_i(x) > 0 \Rightarrow x \in S_i\]
\[g_i(x) < 0 \Rightarrow x \in \overline{S_i}\]

And the decision rule is:
Assign $x$ to class $S_i$ iff $x \in S_i$ and $x \in \overline{S_j}$ for all $j \neq i$.

Draw the design boundaries in feature space, label the classification of regions and identify any indeterminate (undefined) regions. Classify the samples 
$T_{(1)} = (3,4)^T, T_{(2)} = (2,5,2)^T, T_{(3)} = (3,0)^T$.

2. In a multiclass problem, if the samples can be separated by $K(K-1)/2$ hyper plane. Each of which separates a pair of classes, then they are pairwise linearly separable. Let the decision functions for a 3-class problem be 
\[g_{12}(x) = -x_1 - x_2 + 5\]
\[g_{13}(x) = -x_1 + 3\]
\[g_{23}(x) = -x_1 + x_2\]

and $g_{ij}(x) = -g_{ji}(x)$.

The decision rule is:
Assign $x$ to $S_k$ iff $g_{kj}(x) > 0$ for all $j \neq k$.

Draw the decision boundaries and label classified regions and any indeterminate regions. Classify the points $x = (4,3)^T, (0,2)^T$. If there is an indeterminate region prove it by finding a point that doesn’t get classified according to the above rule.
3. Let the discriminate functions for a 3 class problem be
\[ g_1(x) = -x_1 + x_2 \]
\[ g_2(x) = x_1 + x_2 - 1 \]
\[ g_3(x) = -x_2 \]

and the decision rule is the original one described in class:
Assign \( X \) to \( S_k \) iff \( g_k(x) > g_j(x) \) for all \( j \neq k \)

Draw the decision boundaries and label classified regions and any indeterminate regions. Classify the points \( x = (1,1) \). If there is an indeterminate region prove it by finding a point that doesn’t get classified.

4. Show that in general the decision rule of a linear machine (i.e. maximum \( g_k(x) \) as in problem 3) is a special case of the pairwise classifier described in problem 2.

[HINT: express \( g_y(x) \) in terms of \( g_i(x) \) and \( g_j(x) \), as in a 2-class problem].

5. Consider a linear machine with discriminate functions \( g_i(x) = W_i^T X + w_{i0} \), \( i = 1, \ldots, c \).
Show that the decision on regions are convex by showing that if \( X_1 \in R_i \) and \( X_2 \in R_i \) then \( \lambda X_1 + (1-\lambda)X_2 \in R_i \) if \( 0 \le \lambda \le 1 \).