In The Name of God, The Merciful, The Beneficent

Pattern Recognition – CE725
Department of Computer Engineering
Sharif University of Technology
Homework 1 – Fall 2006

Due: 25th of Mehr

1. For a multiclass problem, if any class S_k can be separated from all other classes S_j by a single hyper plane, the samples are totally linearly separable and can be separated by S_k vs. not S_k decisions. Let the decision functions for a 3 class problem be

$g_1(\underline{x}) = -x_1 + x_2$

$g_2(\underline{x}) = x_1 + x_2 - 5$

$g_3(\underline{x}) = -x_2 + 1$

Let

$g_i(\underline{x}) \succ 0 \Rightarrow \underline{x} \in S_i$

$g_i(\underline{x}) \prec 0 \Rightarrow \underline{x} \in \overline{S}_i$

And the decision rule is:

Assign $\underline{x}$ to class S_i iff $\underline{x} \in S_i$ and $\underline{x} \in \overline{S}_j$ for all $j \neq i$.

Draw the design boundaries in feature space, label the classification of regions and identify any indeterminate (undefined) regions. Classify the samples $\underline{x}^{(1)} = (6,5)^T, \underline{x}^{(2)} = (2.5,2)^T, \underline{x}^{(3)} = (3,0)^T$.

2. In a multiclass problem, if the samples can be separated by K(K-1)/2 hyper plane. Each of which separates a pair of classes, then they are pairwise linearly separable. Let the decision functions for a 3-class problem be

$g_{12}(\underline{x}) = -x_1 - x_2 + 5$

$g_{13}(\underline{x}) = -x_1 + 3$

$g_{23}(\underline{x}) = -x_1 + x_2$

and $g_{ji}(\underline{x}) = -g_{ij}(\underline{x})$.

The decision rule is:

Assign $\underline{X}$ to S_k iff $g_{kj}(\underline{x}) \succ 0$ for all $j \neq k$

Draw the decision boundaries and label classified regions and any indeterminate regions. Classify the points $\underline{x} = (4,3)^T, (0,2)^T$. If there is an indeterminate region prove it by finding a point that doesn't get classified according to the above rule.

3. Let the discriminate functions for a 3 class problem be
$g_1(\underline{x}) = -x_1 + x_2$
$g_2(\underline{x}) = x_1 + x_2 - 1$
$g_3(\underline{x}) = -x_2$

and the decision rule is the original one described in class:
Assign $\underline{X}$ to S_k iff $g_k(\underline{x}) \succ g_j(\underline{x})$ for all $j \neq k$

Draw the decision boundaries and label classified regions and any indeterminate regions. Classify the points $\underline{x} = (1,1)^T$. If there is an indeterminate region prove it by finding a point that doesn't get classified.

4. Show that in general the decision rule of a linear machine (i.e. maximum $g_k(\underline{x})$ as in problem 3) is a special case of the pairwise classifier described in problem 2.

[HINT: express $g_{ij}(\underline{x})$ in terms of $g_i(\underline{x})$ and $g_j(\underline{x})$, as in a 2-class problem].

5. Consider a linear machine with discriminate functions $g_i(x) = W_i^t X + w_{i0}, i = 1,....,c$. Show that the decision on regions are convex by showing that if $X_1 \in R_i$ and $X_2 \in R_i$ then $\lambda X_1 + (1-\lambda) X_2 \in R_i$ if $0 \leq \lambda \leq 1$.