HW #2, 2

Chapter 3

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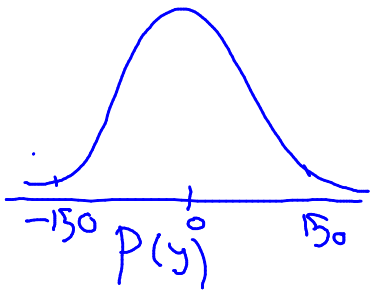
$$P(y) \sim N(\mu, \alpha)$$

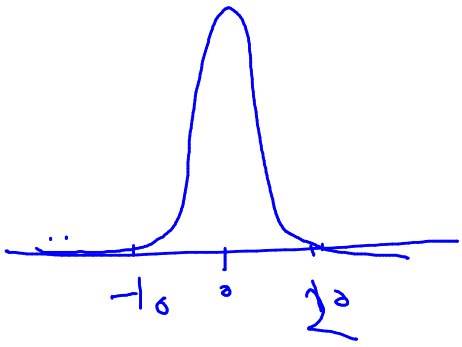
$$P(x|y) \sim N(y, \sigma)$$

$$P(y|x) = \frac{P(y)P(x|y)}{\int_{-\infty}^{+\infty} P(y)P(x|y)\,dy}$$

$$\eta = \frac{\sigma^2\mu + \alpha^2 x}{\sigma^2 + \alpha^2} \qquad \sigma^2 = \frac{\alpha^2\sigma^2}{\alpha^2 + \sigma^2}$$

$$p(y) \times p(x|y) \sim \mathcal{N}\left(\frac{\sigma^2\mu + \alpha^2 x}{\sigma^2 + \alpha^2}, \sqrt{\frac{\alpha^2\sigma^2}{\alpha^2 + \sigma^2}}\right)$$



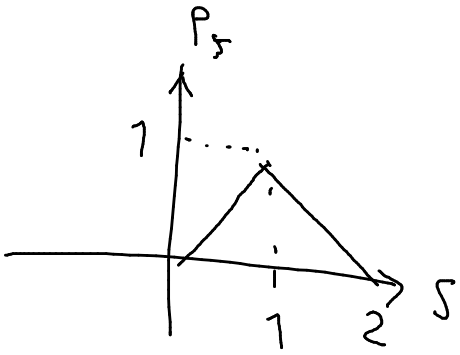


HW #2, 3

$$P_X(x) = P_Y = \begin{cases} 1, & 0 \le x, y \le 1 \\ 0, & \text{otherwise} \end{cases}$$

$$p(s) = \int_{-\infty}^{+\infty} p(s-y)\,p(y)\,dy$$

$$p(s) = \begin{cases} s, & 0 \le s \le 1 \\ 2-s, & 1 < s \le 2 \\ 0, & \text{otherwise} \end{cases}$$



$s < 0 \Rightarrow P(s) = 0$

$s > 2 \Rightarrow P(s) = 0$

$$P(s) = \int_0^s P_1(x)\, P_1(s-x)$$

$0 \le s \le 1 \qquad s - x \ge 0 \Rightarrow s \ge x$

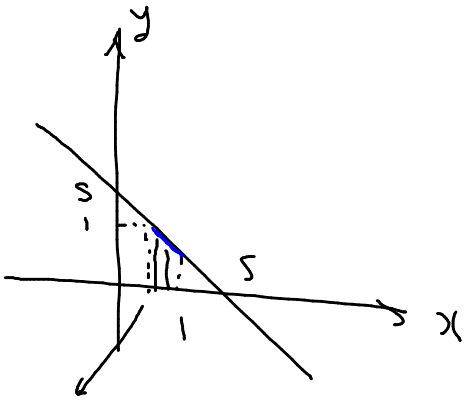
$$P(s) = \int_0^s 1\, dx = s$$

$$1 \le s \le 2$$

$$0 \le s - x \le 1 \Rightarrow s - 1 \le x \le s$$

$$p_s(s) = \int_{s-1}^{1} 1 \, dx = 2 - s$$

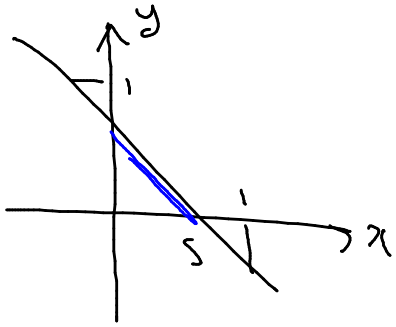
b/ $s > 1$



$x + y = s$

$x = s - 1$

$s < 1$



$$P(x_1 < x < x_2) = \frac{\sqrt{2}(x_2 - x_1)}{\sqrt{2}(2 - s)}$$

$$P(x|s) = \frac{1}{2 - s}, \quad s > 1, \quad s - 1 < x < 1$$

$$P(x|s) = \frac{1}{s}, \quad 0 < x < s$$

$P(x|s) = 0, \quad s > 2$ or $s < 0$

راه حل دوم:

$$p(x|s) = \frac{p(s|x)\,p(x)}{p(s)} = \frac{p(x)\,p(y)}{p(s)}$$

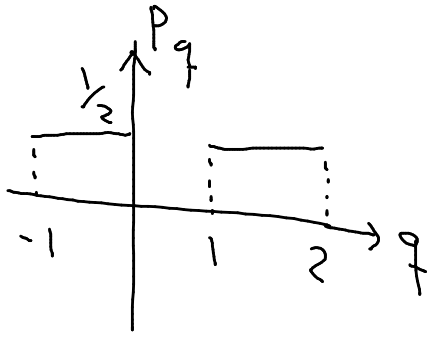
$S = X + Y$

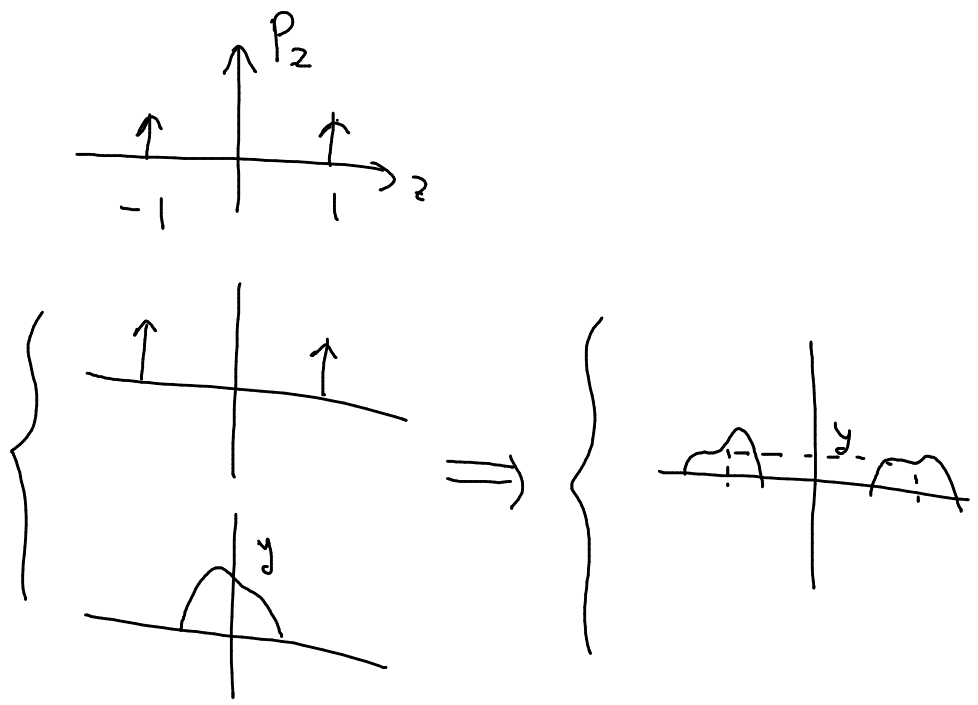
$$p(s|x) = p_s(x+y|x) = p(y)$$

c/

$Z = X + Z$

$$p_q(q) = \int_0^1 p_x(x)\, p_z(q-x)\, dx$$




P_2
z
-1
1
y
y

$$A_x = \begin{bmatrix} 24 & -16 \\ -16 & 24 \end{bmatrix}$$

$A_x e = \lambda e \quad (A_x - \lambda I) = 0 \quad \det(A_x - \lambda I) = 0$

$(24-\lambda)^2 - 256 = 0 \longrightarrow 24 - \lambda = \pm 16 \quad \lambda_1 = 40 \quad \lambda_2 = 8$

$\lambda_1 = 40$

$(A_n - 40I)e = 0 \quad \begin{bmatrix} -16 & -16 \\ -16 & -16 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = 0$

$a = -b$

$\sqrt{a^2 + (-a)^2} = 1$

$e_1 = \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} a \\ -a \end{bmatrix}$

$\lambda_2 = 8$

$(A_n - 8I)e = \begin{bmatrix} 16 & -16 \\ -16 & 16 \end{bmatrix} = 0 \quad a = b$

$e_2 = \begin{pmatrix} 0.7 \\ 0.7 \end{pmatrix}$

$e_1 = \begin{pmatrix} 0.7 \\ -0.7 \end{pmatrix}$