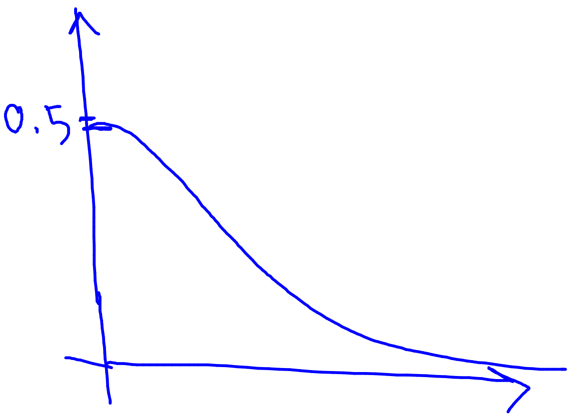
Chapter 2 HW Solutions

2.8) a)

$$P(\text{error}) = \int_{R_2} p(x|\omega_1)\,P(\omega_1)\,dx + \int_{R_1} p(x|\omega_2)\,P(\omega_2)\,dx$$

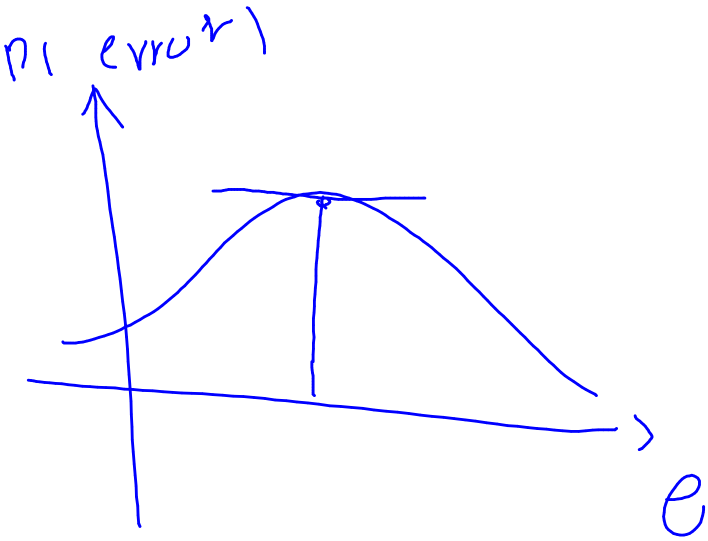
$$= \frac{1}{\pi b}\left[\int_{-\infty}^{\frac{a_2+a_1}{2}} \frac{1}{1+\left(\frac{x-a_1}{b}\right)^2}\,dx + \int_{\frac{a_1+a_2}{2}}^{+\infty} \frac{1}{1+\left(\frac{x-a_2}{b}\right)^2}\,dx\right]$$

$$= \frac{1}{2} - \frac{1}{\pi}\,\operatorname{arctg}\left|\frac{a_2-a_1}{b}\right|$$



$$\frac{a_2 - a_1}{b} \to 0$$

$$a_2 = a_1$$


p(error)
e

(a) $R(\alpha(x) \mid x) = R(\alpha_1 \mid x) P(\alpha_1 \mid x) + \cdots$

(b) $= \sum_{i=1}^{a} R(\alpha_i \mid x) P(\alpha_i \mid x) + R(\alpha_a \mid x) P$

$$R = \int R(\alpha(x) \mid x) P(x) \, dx$$

$$R(\alpha_{i^*}|x) \leq R(\alpha_i|x) \quad \text{for all } i \neq i^*$$

$$R(\alpha(x)|x) = \sum_{i=1}^{a} R(\alpha_i|x) P(\alpha_i|x) =$$

$$= \sum_{i=1}^{a} \left[R(\alpha_i|x) - R(\alpha_{i^*}|x) + R(\alpha_{i^*}|x) \right] P(\alpha_i|x)$$

$$= R(\alpha_{i^*}|x) \sum_{i=1}^{a} P(\alpha_i|x) + \sum_{i \neq i^*} \left(R(\alpha_i|x) - R(\alpha_{i^*}|x) \right) P(\alpha_i|x)$$

$$= R(\alpha_{i^*}|x) + \sum \left(R(\alpha_i|x) - R(\alpha_{i^*}|x) \right)$$

$\vdots$

Because $R(\alpha_i|x) - R(\alpha_{i^*}|x) \geq 0$

$\Rightarrow$ Maximize can be achieved by

$P(\alpha_i|x) = 0$ for $i \neq i^*$

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$$R(\alpha_i|x) = \sum_{j=1, j\neq i}^{c} \lambda_s P(\omega_j|x)$$
$1 \leq i \leq c$

$$= \lambda_s \sum_{j=1, j\neq i}^{c} P(\omega_j|x) = \lambda_s (1 - P(\omega_i|x))$$

$$R(\alpha_{c+1}|x) = \lambda_r$$

$$i \begin{cases} \lambda_s(1 - P(\omega_i|x)) \leq \lambda_s(1 - P(\omega_j|x)) \\ \lambda_s(1 - P(\omega_i|x)) \leq \lambda_r \end{cases}$$

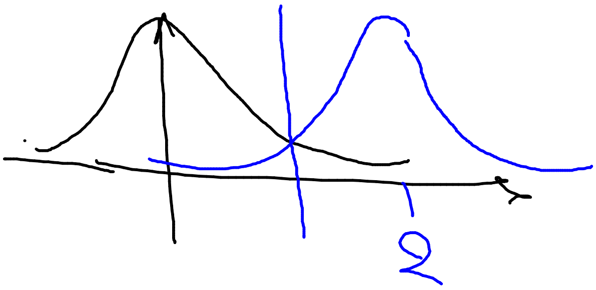
$$i \begin{cases} p(\omega_i|x) \geq p(\omega_j|x) \\ p(\omega_i|x) \geq 1 - \dfrac{\lambda_r}{\lambda_s} \end{cases}$$

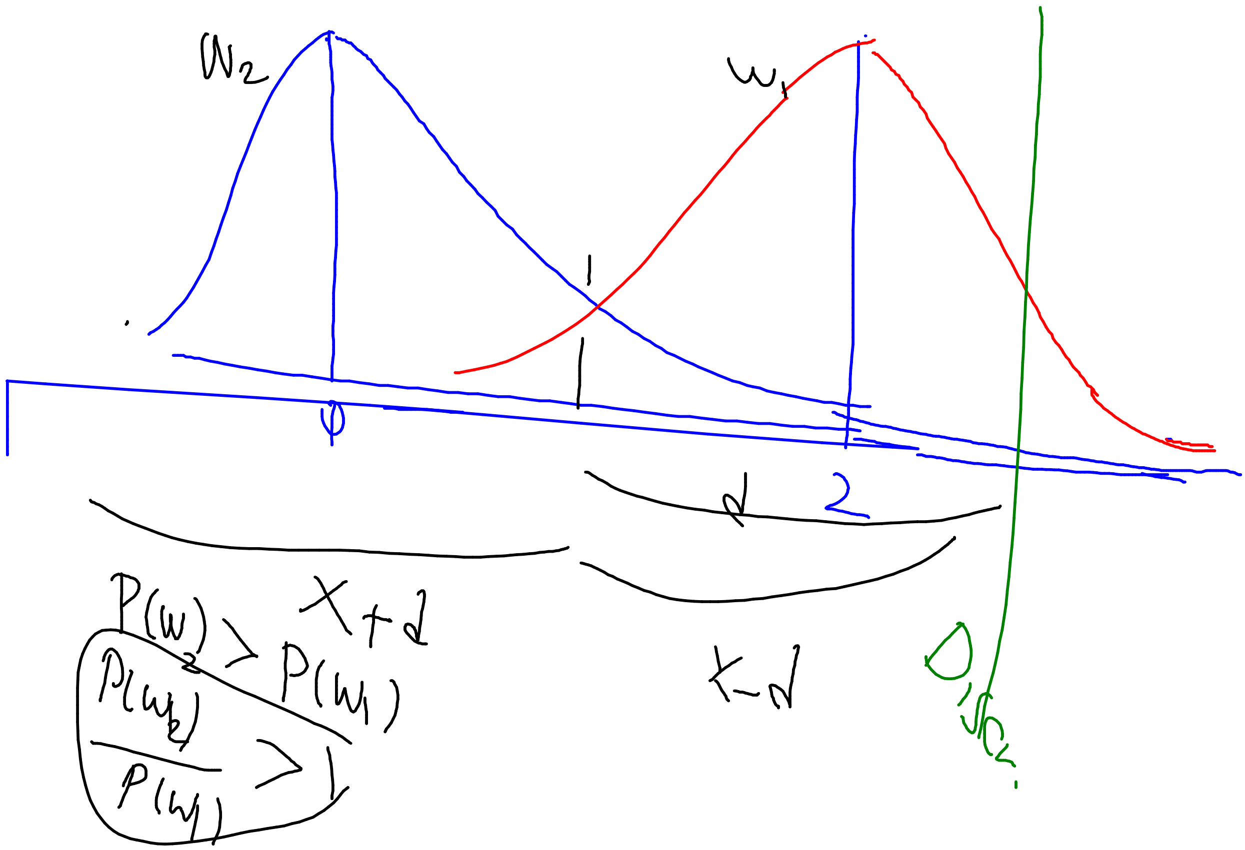
2-27 $\qquad \mu_1 > \mu_2$

$$(63) \Rightarrow x_0 = \frac{1}{2}(\mu_1 + \mu_2) - \frac{\ln\left(\frac{P(\omega_1)}{P(\omega_2)}\right)}{(\mu_1 - \mu_2)^t \Sigma^{-1} (\mu_1 - \mu_2)} (\mu_1 - \mu_2)$$

$x_0 > \mu_1$

$$\frac{P(\omega_1)}{P(\omega_2)} < \exp\left(-\frac{1}{2}(\mu_1 - \mu_2)^t \Sigma^{-1} (\mu_1 - \mu_2)\right)$$





w_2
w_1
1
φ
d
2
x+d
x−d
P(w_2) > P(w_1)
$\frac{P(w_2)}{P(w_1)} > 1$

$P(\omega_k | \underline{x}) > P(\omega_j | \underline{x})$ for all j and k

$\Rightarrow \underline{x} \in S_k$

$$P(\omega_j | \underline{x}) = P(\underline{x} | \omega_j) P(\omega_j)$$

$$= \prod_{i=1}^{d} P(x_i | \omega_j) P(\omega_j)$$

$$= \prod_{i=1}^{d} (P_{ij})^{x_i} (1 - P_{ij})^{1-x_i} \; P(\omega_j)$$

$$= (P_{ij})^{\sum x_i} (1 - P_{ij})^{\sum(1-x_i)} \; P(\omega_j)$$

$$\Rightarrow \ln(P(\omega_j|x)) = \sum x_i \ln P_{ij} + \ldots$$

$$\ldots \sum (1-x_i) \ln(1-P_{ij}) + \ln P(\omega_j)$$

$$= \sum_{i=1}^{d} x_i \left(\ln P_{ij} - \ln(1 - P_{ij}) \right) + \ldots$$

$$\sum \ln(1 - P_{ij}) + \ln P(\omega_j)$$

$$\Rightarrow g_j(x) = \sum_{i=1}^{d} x_i \ln\left(\frac{P_{ij}}{1 - P_{ij}} \right) + \sum_{i=1}^{d} \ln(1 - P_{ij})$$

$$+ \ln(P(\omega_j)) \quad \text{QED.}$$