1. There are $m$ balls, labeled 1,2,\ldots,m in two boxes, A and B. At step $n$, one chooses uniformly and independently of the history, an integer $I_n$ in the set $\{1, \ldots, m\}$, and moves the $I_n$-ball from the box where it is to the other box. Let $X_n$ denote the number of balls in the A box after $n$ steps. Prove that $X_n$ is a time-homogeneous Markov process, determine its transition probability matrix, classify its states, and find its stationary distribution when $m = 3$.

2. The following is a model of cell splitting: at time $n$, a living cell is split into two cells with probability $p$, or dies with probability $1-p$. The splitting of different cells is an independent sequence. Let $Z_n$ be the number of living cells at time $n$. Assume $Z_0 = 2$. Prove that $\{Z_n\}$ is a time-homogeneous Markov process, determine its state space and its transition matrix, find its stationary distribution(s), and classify the states as transient or recurrent.

3. Three tanks fight a three-way duel. Tank A has probability 1/2 of destroying the tank at which it fires, tank B has probability 1/3 of destroying the tank at which it fires, and tank C has probability 1/6 of destroying the tank at which it fires. The tanks fire together and each tank fires at the strongest opponent not yet destroyed. Form a Markov chain by taking as states the subsets of the set of tanks. Find the expected number of steps before the chain is absorbed.