Assignment 4

1. Give state diagrams of DFAs recognizing the following languages. In all cases the alphabet is \{0,1\}.
   \[ L_1 = \{ w \mid \text{w doesn’t contain the substring 110} \} \]
   \[ L_2 = \{ w \mid \text{the length of w is at most 5} \} \]
   \[ L_3 = \{ w \mid \text{w contains at least two 0s and at most one 1} \} \]
   \[ L_4 = \{ w \mid \text{every odd position of w is a 1} \} \]
   \[ L_5 = \{ w \mid \text{w is any string except 11 and 111} \} \]

2. Give state diagrams of DFAs recognizing the following languages. The alphabet is \{a,b\}.
   \[ L_1 = \{ w \mid |n_a(w) - n_b(w)| \mod 3 < 2 \} \]
   \[ L_2 = \{ w \mid |n_a(w) - n_b(w)| \mod 3 = 2 \} \]
   \[ L_3 = \{ w \mid |n_a(w) - n_b(w)| \mod 3 > 2 \} \]

3. Show that, if M is a DFA that recognizes language B, swapping the accept and non-accept states in M yields a new DFA that recognizes the complement of B. Conclude that the class of regular languages is closed to under complement.

4. Give grammar of these languages.
   \[ L_1 = \{ a^n b^n : n \geq 0, M > n \} \]
   \[ L_2 = \{ a^n b^{2n} : n \geq 0 \} \]
   \[ L_3 = \{ a^n b^{n-\frac{1}{2}} : n \geq 3 \} \]
   \[ L_4 = L_1 L_2 \]
   \[ L_5 = L_1^* \]

5. Give NFAs with the specified number of states recognizing each the following languages.
   * the language \{w \mid w ends with 000\} with three states
   * the language \{0^* 1^* 0^* 0\} with three states
   * the language \{0\} with two states

6. Convert the following two nondeterministic finite automata to equivalent deterministic finite automata.