Homework 2 (Chapters 1 & 2)

Problems

1. Determine and sketch the following signal and its even and odd parts. Label your sketches carefully.
   \[ x(t) = u(t + 1) - u(t) + (u(t) - u(t - 1)) \ast (u(t - 1) - u(t - 2)) \ast \delta(t - 1) \]

2. Are the following signals periodic? If so determine their fundamental period.
   a. \( w(t) = \cos(\frac{\pi}{4}t) + \cos(\frac{3\pi}{5}t) \)
   b. \( z(t) = \sin(\frac{\pi}{3}t) + \cos(t) \)

3. A system may or may not be linear, time-invariant, memoryless, causal, or stable. Determine whether or not each of the following systems has these properties.
   a. \( y(t) = \sum_{i=-5}^{5} a_i x(t - i) \)
   b. \( y[n] = \begin{cases} (-1)^n x[n] & \text{if } x[n] \geq 0 \\ 2x[n] & \text{if } x[n] < 0 \end{cases} \)
   c. \( y[n] = \text{Odd}\{x[n - 1]\} \)

4. Consider an LTI system whose response to the signal \( x_1(t) \) is the signal \( y_1(t) \) where these signals are depicted below. Determine and provide a labeled sketch of the response to the input \( x_2(t) \), which is also depicted below.

5. Compute the convolution sum \( y[n] = x[n] \ast h[n] \) for each of the following pairs of signals.
   a. \( x[n] = h[n] = \alpha^n u[n] \)
   b. \( x[n] = (-\frac{1}{4})^n u[n - 4], h[n] = 2^n u[3 - n] \)

6. Compute the convolution \( y(t) = x(t) \ast h(t) \) for each of the following pairs of signals.
   a. \( x(t) = h(t) = e^{-\alpha t} u(t) \)
b. \( x(t) = x_1(t) \) (from Prob. 6), \( h(t) = (u(t) - u(t - 1)) * (u(t) - u(t - 1)) \)

7. The following are the impulse responses of LTI systems. Determine whether each system is causal and/or stable. Justify your answers. (P 2.28 e,g and 2.29 e,f)

a. \( h[n] = (-\frac{1}{2})^n u[n] + (1.01)^n u[n - 1] \)
b. \( h[n] = n(\frac{1}{3})^n u[n - 1] \)
c. \( h(t) = e^{-6|t|} \)
d. \( h(t) = te^{-t} u(t) \)

8. P 2.40 p. 148
9. P 2.47 p. 152
10. P 2.64 a,b,d p. 166

11. Let \( \mathbf{V} \) be a subspace of Hilbert space \( \mathbf{H} \). A projector \( P_{\mathbf{V}} \) on \( \mathbf{V} \) is a linear operator that satisfies:

1. \( \forall f \in \mathbf{H} \rightarrow P_{\mathbf{V}} f \in \mathbf{V} \)
2. \( \forall f \in \mathbf{V} \rightarrow P_{\mathbf{V}} f = f \)

The projector \( P_{\mathbf{V}} \) is orthogonal if \( \forall f \in \mathbf{H}, \forall g \in \mathbf{V} \rightarrow \langle f - P_{\mathbf{V}} f, g \rangle = 0 \). If \( P_{\mathbf{V}} \) is an orthogonal projector on \( \mathbf{V} \), prove that:

a. \( \forall f \in \mathbf{H} \rightarrow ||f - P_{\mathbf{V}} f|| = \min_{g \in \mathbf{V}} ||f - g|| \).
b. If \( \{e_n\}_n \in \mathcal{N} \) is an orthogonal basis of \( \mathbf{V} \), then \( P_{\mathbf{V}} f = \sum_{n=0}^{\infty} \frac{\langle f, e_n \rangle}{||e_n||^2} e_n \)

Practical Assignment

1. Define the MATLAB vector \( \mathbf{x} \) to be the values of the signal \( x[n] \) at those samples, where \( x[n] \) is given by

\[
x[n] = \begin{cases} 
2, & n = 0, \\
1, & n = 2, \\
-1, & n = 3, \\
3, & n = 4, \\
0, & \text{otherwise}
\end{cases}
\]

If you define these vectors correctly you should be able to plot this discrete-time sequence using \texttt{stem(nx,x)}. Now plot \( x[n-2] \) and \( x[-n+1] \).

2. Consider the impulse response \( h[n] = \delta[n+1] + \delta[n-1] \) and the input \( x[n] = \delta[n] - 3\delta[n-2] \). In MATLAB define the vectors \( \mathbf{h} \) and \( \mathbf{x} \) corresponding to these sequences. Use \texttt{conv} (read help for \texttt{conv: help conv}) to compute the output signal \( y[n] \). Determine a vector of time indices corresponding to \( y \) and store those in the vector \( \mathbf{y} \). Plot \( y[n] \) as a function of \( n \) using the command \texttt{stem(ny,y)}.

3. Consider the impulse response \( h[n] \) and input \( x[n] \) defined below

\[
h[n] = u[n+2] \\
x[n] = (\frac{2}{3})^n - 2u[n-2]
\]

a. Analytically compute and sketch the convolution of \( h[n] \) and \( x[n] \).
b. Define a vector \( h \) in MATLAB that contains the values of \( h[n] \) for \( 2 \leq n \leq 14 \), and define a vector \( x \) in MATLAB that contains the values of \( x[n] \) for \( 0 \leq n \leq 24 \). Define vectors \( nh \) and \( nx \) containing the corresponding time indices. Compute the convolution of these two finite-length vectors using \( y = \text{conv}(x, h) \). Compute the vector of time indices for \( y \) and store that in \( ny \). Plot \( y \) using the \text{stem} command. Note that since the \( h \) and \( x \) vectors are shortened versions of the true signals, only a portion of the output vector will contain the true values of \( y[n] \). Specify which values in the output vector \( y \) are correct and which are not. Explain your answer.