Homework 4 (Chapter 4)

Problems

1. Compute the Fourier transform of each of the following signals:
   a. \(x(t) = e^{-3|t|} \sin 2t\)
   b. \([te^{-2t} \sin 4t]u(t)\)
   c. \(x(t) = \begin{cases} \frac{b^2}{2}(1 - \frac{|t|}{2a}), & |t| < 2a \\ 0, & \text{o.w.} \end{cases}\), Where \(a, b > 0\).
   d. \(\frac{\sin \pi t}{\pi t} \left[\frac{\sin 2\pi (t-1)}{\pi(t-1)}\right]\)
   e. \(x(t) = \begin{cases} 1 - t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}\)
   f. \(x(t)\) as shown in Figure P4.21(b) in p. 338 of textbook.

2. Determine the continuous-time signal corresponding to each of the following transforms:
   a. \(X(j\omega) = j[\delta(\omega + 1) - \delta(\omega - 1)] - 3[\delta(\omega - \pi) + \delta(\omega + \pi)]\)
   b. \(X(j\omega) = \frac{2\sin[3(\omega - 2\pi)]}{\omega - 2\pi}\)
   c. 

   ![Plot 1](image1.png)

   ![Plot 2](image2.png)

   d. 

   ![Plot 3](image3.png)

   ![Plot 4](image4.png)

3. Determine which, if any, of the real signals depicted in below have Fourier transforms that satisfy each of the following conditions:
   1. \(\Re\{X(j\omega)\} = 0\)
2. $\text{Im}\{X(j\omega)\} = 0$

3. There exists a real $\alpha$ such that $e^{j\alpha \omega}X(j\omega)$ is real

4. $\int_{-\infty}^{\infty} X(j\omega)d\omega = 0$

5. $\int_{-\infty}^{\infty} \omega X(j\omega)d\omega = 0$

6. $X(j\omega)$ is periodic

4. Let $X(j\omega)$ represent the Fourier transform of

$$x(t) = \begin{cases} 
e^{-t} & 0 < t < 1 \\ 0 & \text{o.w} \end{cases}$$

Express the Fourier Transforms of each of the following signals in terms of $X(j\omega)$.
5. Suppose an LTI system is described by the following LCCDE:

\[
\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + 3y(t) = 4\frac{dx(t)}{dt} - x(t)
\]

Show that \(Y(j\omega)\) can be expressed as \(Y(j\omega) = H(j\omega)X(j\omega)\) and find \(H(j\omega)\).

6. Let \(x_1(t)\) represent the input to an LTI system, where

\[
x_1(t) = \sum_{k=-\infty}^{+\infty} \alpha^{|k|} e^{jk\pi t}
\]

for \(0 < \alpha < 1\). The frequency response of the system is

\[
H(j\omega) = \begin{cases} 
1 & |\omega| < W \\
0 & \text{otherwise}
\end{cases}
\]

What is the minimum value of \(W\) so that the average energy in the output signal will be at least 90% of that in the input signal.

7. Consider the impulse train

\[
p(t) = \sum_{k=-\infty}^{+\infty} \delta(t - kT)
\]
(a) Find the Fourier series of \( p(t) \).
(b) Find the Fourier transform of \( p(t) \).
(c) Consider the signal \( x(t) \) shown in following figure, where \( T_1 < T \).

\[
\begin{align*}
x(t) \quad &
\end{align*}
\]

Show that the periodic signal \( \tilde{x}(t) \), formed by periodically repeating \( x(t) \), satisfies

\[
\tilde{x}(t) = x(t) * p(t)
\]

(d) find the Fourier transform of \( \tilde{x}(t) \) in terms of the Fourier transform of \( x(t) \).

8. Suppose that the system \( F \) takes the Fourier transform of the input, as shown:

\[
\begin{align*}
x(t) \quad &\quad \text{F} \quad \text{y}(t) = 2\pi X(-\omega) \bigg| \omega = \omega_0 \\
\end{align*}
\]

What is \( w(t) \) calculated in following figure:

\[
\begin{align*}
x(t) \quad &\quad \text{F} \quad \text{F} \quad \text{F} \quad w(t)
\end{align*}
\]

9. (a) Show that the three LTI systems with impulse responses

\[
\begin{align*}
h_1(t) &= u(t), \\
h_2(t) &= -2\delta(t) + 5e^{-2t}u(t), \\
h_3(t) &= 2te^{-t}u(t)
\end{align*}
\]

all have the same response to \( x(t) = \cos(t) \).

(b) Find the impulse response of another LTI system with the same response to \( \cos(t) \).

This problem illustrates the fact that the response to \( \cos(t) \) cannot be used to specify an LTI system uniquely.