Homework 8

Problems

1. Determine the Laplace transforms (including the regions of convergence) of each of the following signals.
   a. \( x[n] = (\frac{1}{2})^n u[n - 3] \)
   b. \( x[n] = (1 + n)(\frac{1}{3})^n u(n) \)
   c. \( x[n] = n(\frac{1}{2})^{|n|} \)

2. Determine the inverse z-transform of each of the following.
   a. \( X(Z) = \frac{1}{z-1} \)
   b. \( X(Z) = \frac{1}{z^2 + z + 1} \)
   c. \( X(Z) = \frac{2z^{-3}}{(1 - \frac{1}{4}z^{-1})^2}, \) \( x[n] \) is left sided.
   d. \( X(Z) = \sin(z), \) ROC includes \(|z| = 1.\)
   e. \( X(Z) = \frac{z^7 - 2}{1 - z^{-7}}, \) \(|z| > 1.\)
   f. \( X(Z) = e^z + e^{1/z}, z \neq 0 \)

3. Determine the unit step response of the causal system for which the z-transform of the impulse response is:

\[
H(z) = \frac{1 - z^3}{1 - z^4}
\]

4. If the input \( x[n] \) to an LTI system is \( x[n] = u[n], \) the output is

\[
y[n] = (\frac{1}{2})^{n-1} u[n + 1]
\]

   a. Find \( H(z), \) the z-transform of the system impulse response, and plot its pole-zero diagram.
   b. Find the impulse response \( h[n].\)
c. Is the system stable.
d. Is the system casual.

5. Consider the following DT pole-zero diagrams, where the circles have unit radius.

![Diagram with pole-zero plots]

a. Which if any of the pole-zero plots could represent the z-transform of the following DT signal?

![signal diagram]

b. Which if any of the pole-zero diagrams could represent a system that is stable?
c. Which if any of the pole-zero diagrams could represent a system that is casual?
d. Which if any of the pole-zero diagrams could represent a system that is both stable and casual?

6. Consider an LTI system with input \( x[n] \) and output \( y[n] \) that satisfies the difference equation

\[
y[n] - \frac{2}{5}y[n - 1] + y[n - 2] = x[n] - x[n - 1]
\]

Determine all possible values for the system's impulse response \( h[n] \) at \( n = 0 \).

7. Consider the digital filter structure shown below.

![Filter structure diagram]

a. Find \( H(z) \) for this casual filter. Plot the pole-zero pattern and indicate the region of convergence.
b. For what value of \( k \) is the system stable.
c. Determine \( y[n] \) if \( k = 1 \) and \( x[n] = (\frac{2}{3})^n \) for all \( n \).