Constraint Satisfaction Problems

CE417: Introduction to Artificial Intelligence
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In *which we see how treating states as more than just little black boxes leads to the invention of a range of powerful new search methods and a deeper understanding of problem structure and complexity*

Course material: “Artificial Intelligence: A Modern Approach”, 3rd Edition, Chapter 6

Outline

- Constraint Satisfaction Problems (CSP)
  - Representation for wide variety of problems
  - CSP solvers can be faster than general state-space searchers
- Backtracking search for CSPs
- Inference in CSPs
- Local search for CSPs
- Problem Structure

Constraint Satisfaction Problems (CSPs)

- Standard search problem:
  - State is a "black box" with no internal structure (it is a goal or not a goal)

- Solving CSPs more efficiently
  - State is specified by variables or features X_i $(i = 1, \ldots, n)$ (factored representation)
  - Goal test: Whether each variable has a value that satisfies all the constraints on the variable?

- CSPs yield a natural representation for a wide variety of problems

- CSP search algorithms use general-purpose heuristics based on the structure of states
  - Eliminating large portions of the search space by identifying variable/value combinations that violate the constraints

What is CSPs?

- In CSPs, the problem is to search for a set of values for the variables (features) so that the assigned values satisfy constraints.

What is CSPs?

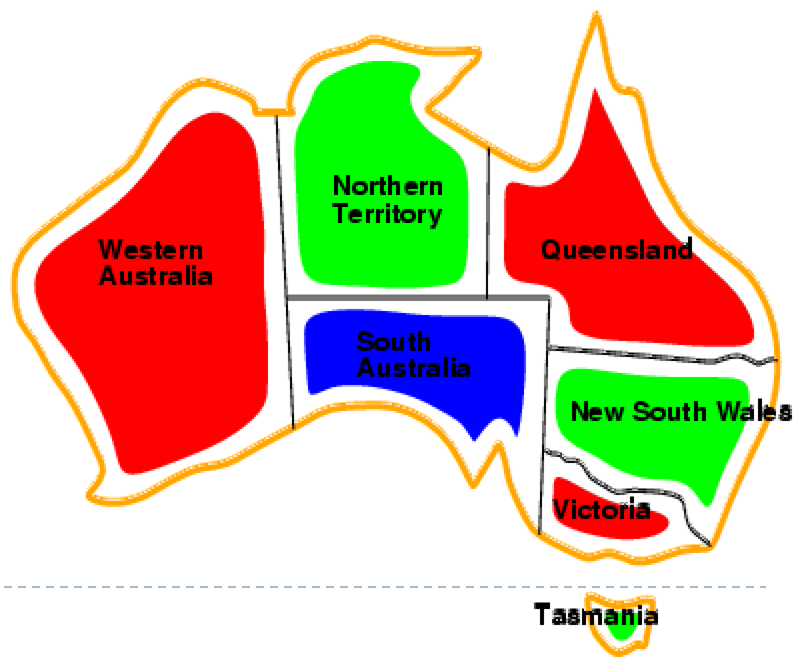
- Components of a CSP
  - X is a set of variables $\{X_1, X_2, \dots, X_n\}$
  - D is the set of domains $\{D_1, D_2, \dots, D_n\}$ where D_i is the domain of X_i
  - C is a set of constraints $\{C_1, C_2, \dots, C_m\}$
    - Each constraint limits the values that variables can take (e.g., $X_1 \neq X_2$)

- Solving a CSP
  - State: An assignment of values to some or all of the variables
  - Solution (goal): A <u>complete</u> and <u>consistent</u> assignment
    - Consistent: An assignment that does not violate any constraint
    - Complete: All variables are assigned.

CSP: Map coloring example

- Coloring regions with tree colors such that no neighboring regions have the same color
  - Variables corresponding to regions: $X = \{WA, NT, Q, NSW, V, SA, T\}$
  - The domain of all variables is $\{red, green, blue\}$
  - Constraints: $C = \{SA \neq WA, SA \neq NT, SA \neq Q, SA \neq NSW, S \neq V,$
    $WA \neq NT, NT \neq Q, Q \neq NSW, NSW \neq V\}$

- A solution:
  $\{WA = red, NT = green, Q = red, NSW = green, V = red, T = green\}$



Example: N-Queens

- Variables: $\{Q_1, Q_2, \ldots, Q_N\}$
- Domains: $\{1,2, \ldots, N\}$
- Constraints:
  - Implicit: $\forall i, j \neq i \;\; non\_threatening(Q_i, Q_j)$
  - Explicit: $(Q_i, Q_j) \in \{(1,3), (1,4), \ldots, (8,6)\}$

Boolean satisfiability example

- Given a Boolean expression, is it satisfiable?
  - $e.g., ((p_1 \land p_2) \rightarrow p_3) \lor (\neg p_1 \land p_3)$
- Representing Boolean expression in 3-CNF
- 3-SAT: find a satisfying truth assignment for 3-CNF
  - Variables: $p_1, p_2, \ldots, p_n$
  - Domains: $\{true, false\}$
  - Constraints: *all clauses must be satisfied*

Real world examples of CSPs

- Assignment problems
  - e.g., who teaches what class?
- Timetabling problems
  - e.g., which class is offered when and where?
- Transportation scheduling
- Factory scheduling
  - Job Shop Scheduling

Types of variables in CSP formulation

- Discrete variables
  - Finite domains
    - n variables and $|D_i| = d \ (i = 1, \dots, n) \Longrightarrow O(d^n)$ complete assignments
  - Infinite domains
    - e.g., job scheduling, variables are start/end days for each job
      - A constraint language is required (e.g., $StartJob_1 + d_1 \leq StartJob_2$)

- Continuous variables
  - e.g., exact start/end times for Hubble Space Telescope observations

Types of constraints in CSP formulation

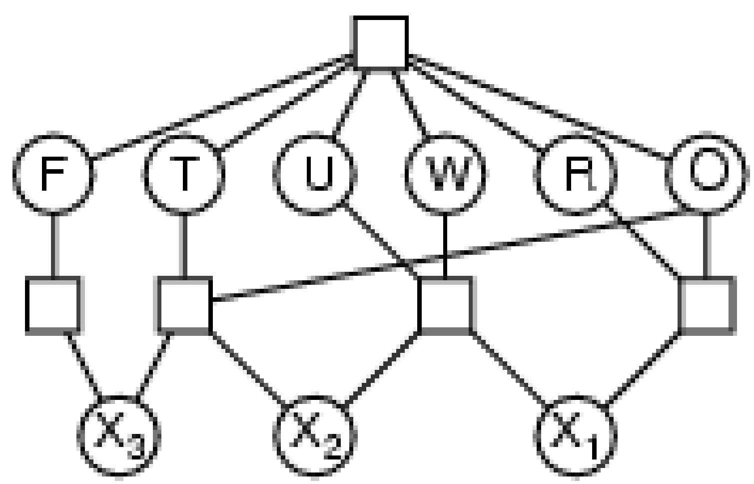
- Unary constraints involve a single variable
  - e.g., $SA \neq green$
- Binary constraints involve pairs of variables
  - e.g., $SA \neq WA$
- Higher-order constraints involve 3 or more variables
  - e.g., cryptarithmetic column constraints
  - Every higher-order finite constraint can be broken into *n* binary constraints, given enough auxiliary constraints
- Preference (soft constraints) $\Longrightarrow$ Constraint optimization problem
  - e.g., red is better than green often representable by a cost for each variable assignment

Cryptarithmetic example

- **Variables**: $X = \{F, T, U, W, R, O, X_1, X_2, X_3\}$
- **Domains**: $\{0,1,..,9\}$
- **Constraints**:
  - $alldiff(F, T, U, W, R, O)$ → Global constraint
  - $O + O = R + 10 \times X_1$
  - $X_1 + W + W = U + 10 \times X_2$
  - $X_2 + T + T = O + 10 \times X_3$
  - $X_3 = F,\ T \neq 0,\ F \neq 0$

```
    T W O
  + T W O
  -------
  F O U R
```

Hypergraph



Sudoku example

- Variables: Each (open) square
- Domains: {1,2, … , 9}
- Constraints:
  - 9-way $alldiff$ for each row
  - 9-way $alldiff$ for each column
  - 9-way $alldiff$ for each region

	1	2	3	4	5	6	7	8	9
A			3		2		6		
B	9			3		5			1
C			1	8		6	4		
D			8	1		2	9		
E	7								8
F			6	7		8	2		
G			2	6		9	5		
H	8			2		3			9
I			5		1		3		

(a)

	1	2	3	4	5	6	7	8	9
A	4	8	3	9	2	1	6	5	7
B	9	6	7	3	4	5	8	2	1
C	2	5	1	8	7	6	4	9	3
D	5	4	8	1	3	2	9	7	6
E	7	2	9	5	6	4	1	3	8
F	1	3	6	7	9	8	2	4	5
G	3	7	2	6	8	9	5	1	4
H	8	1	4	2	5	3	7	6	9
I	6	9	5	4	1	7	3	8	2

(b)

Solving CSPs as a standard search problem Incremental

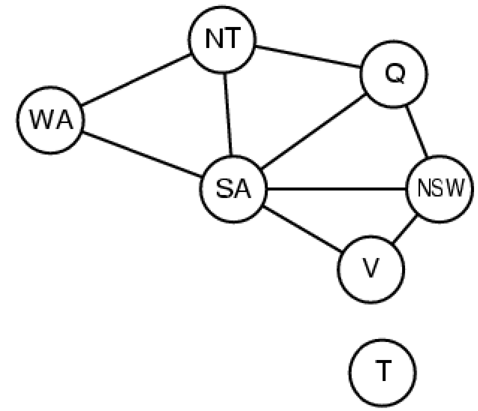
- **Initial State:** { }
- **Actions:** assign a value to an unassigned variable that does not conflict with current assignment
- **Goal test:** Consistent & complete assignment
- **Path cost:** 1 for each action

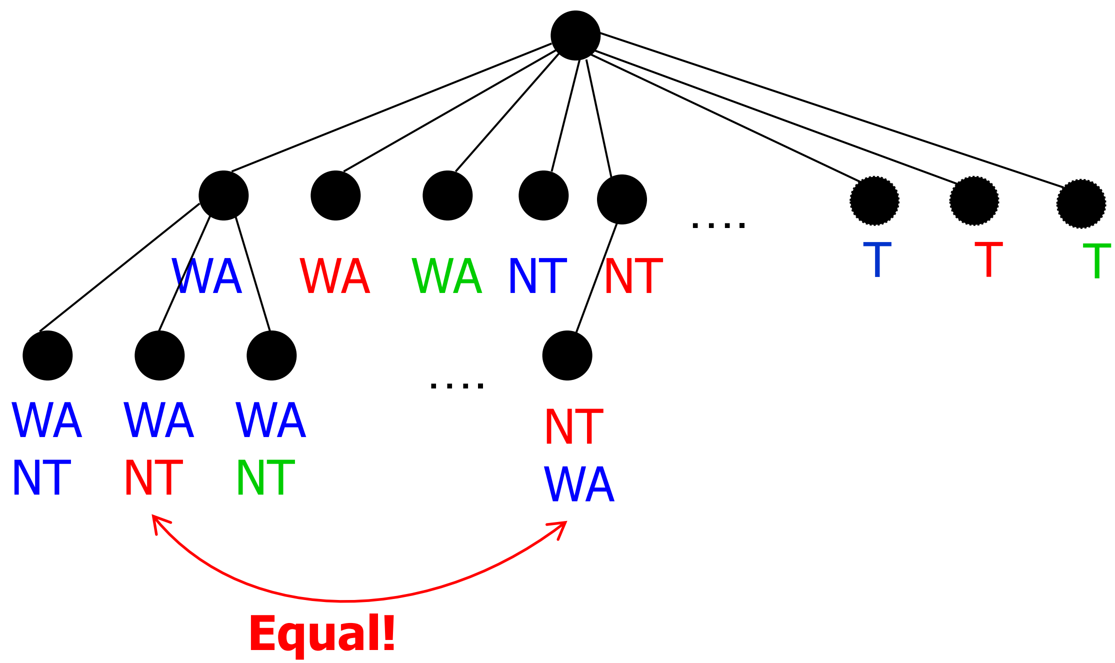
Properties of CSPs as a standard search problem

- Generic problem formulation: same formulation for all CSPs
- Every solution appears at depth n with n variables
- Branching factor is nd at the top level, $b = (n - l)d$ at depth l, hence there are $n!\,d^n$ leaves.
  - However, there are only d^n complete assignments.
- Which search algorithm is proper?
  - DFS

Assignment community

- When assigning values to variables, we reach the same partial assignment regardless of the order of variables





$n \times d$

$(n \times d) \times ((n-1) \times d)$

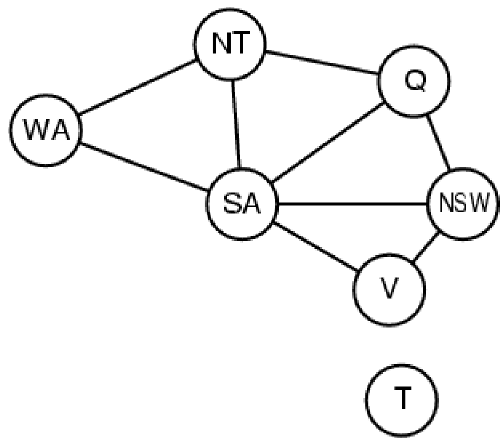
…

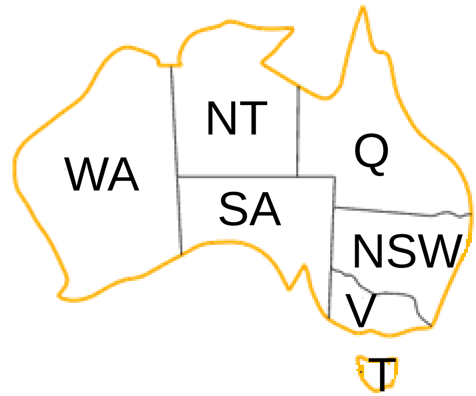
$n! \times d^n$

There are $n! \times d^n$ leaves in the tree but only d^n distinct states!

Constraint graph

- Nodes are variables, arcs are constraints





- Enforcing local consistency in each part of the graph can cause inconsistent values to be eliminated
  - Example: map coloring, $SA = blue$
    - 2^5 instead of 3^5 assignments for neighboring variables

- Using the graph structure can speed up the search process
  - e.g., T is an independent sub-problem

CSPs solvers can be fast

- Many intractable problems for regular state-space search, can be solved quickly by formulating as CSPs using these techniques:
  - Backtracking
    - when a partial assignment is found inconsistent, discards further refinements of this (not promising) assignment.
  - Ordering heuristics
    - order of variables to be tried and selection of values to be assigned.
  - Domains reduction
    - eliminates values causing future failure from domains of variables (before or during search).
  - Graph structure
    - Decompose the whole problem into disjoint or least connected sub-problems

CSPs solver phases

- Combination of <u>combinatorial search</u> and <u>heuristics</u> to reach reasonable complexity:
  - Search
    - Select a new variable assignment from several possibilities of assigning values to unassigned variables
  - Inference in CSPs (constraint propagation)
    - "<u>looking ahead</u>" in the search at unassigned variables to eliminate some possible part of the future search space.
      - Using the constraints to reduce legal values for variables

CSPs solver phases

- Search
  - Base of the search process is a backtracking algorithm
- Inference in CSPs (constraint propagation)
  - Key idea is local consistency

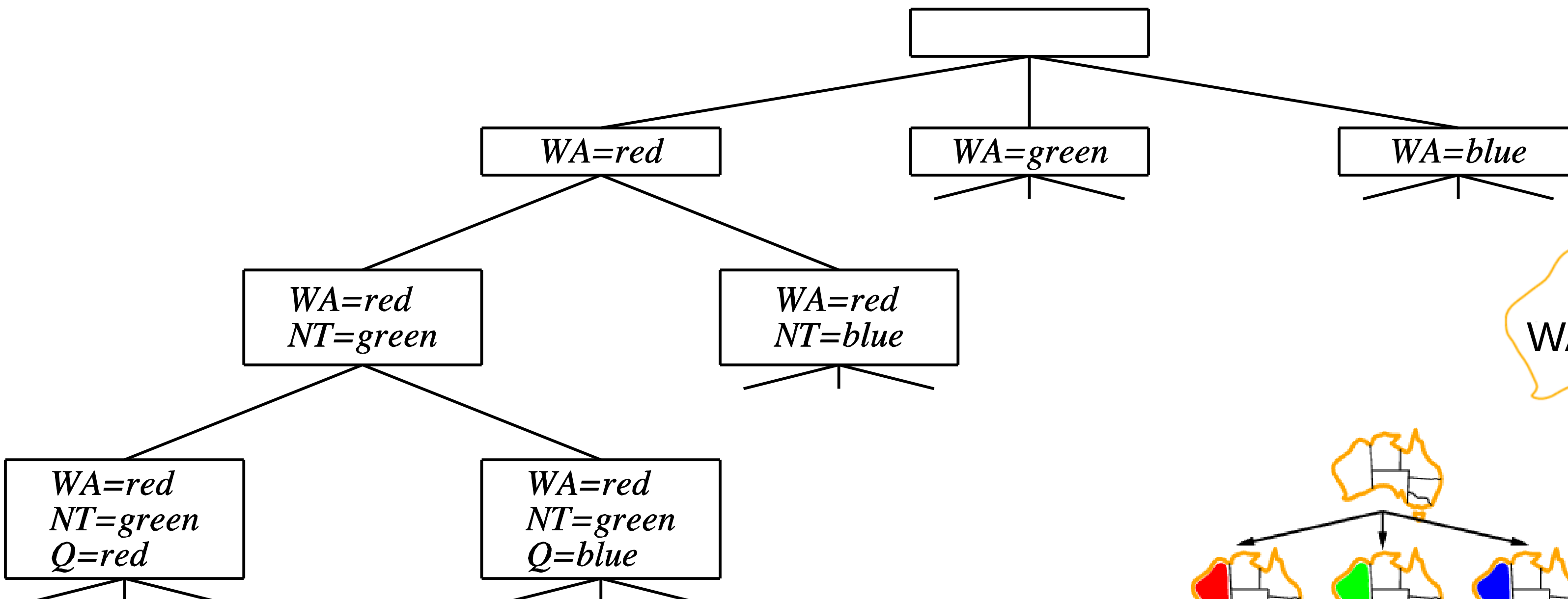
Backtracking search

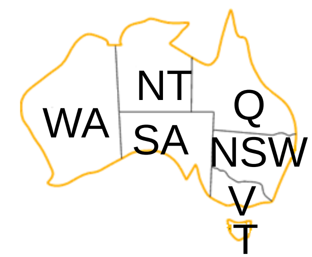
- Depth-first search for CSPs with single-variable assignments is called backtracking search
  - assigns one variable at each level (eventually they all have to be assigned.)

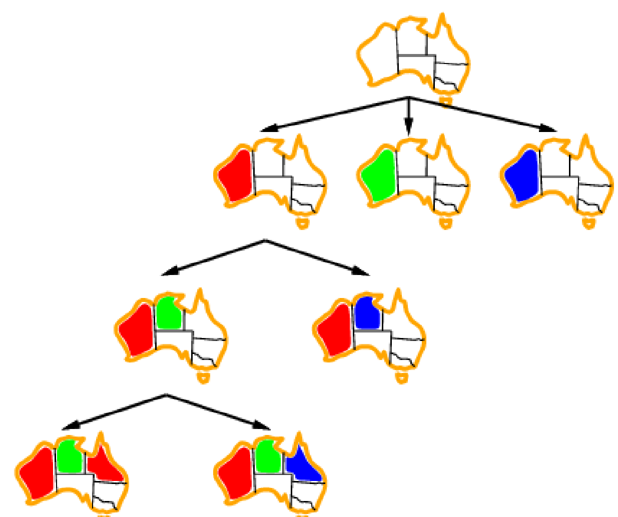
- Naïve backtracking is not generally efficient for solving CSPs.
  - More heuristics will be introduced later to speedup it.

Search tree

- Variable assignments in the order: $WA, NT, Q, \ldots$







Backtracking search

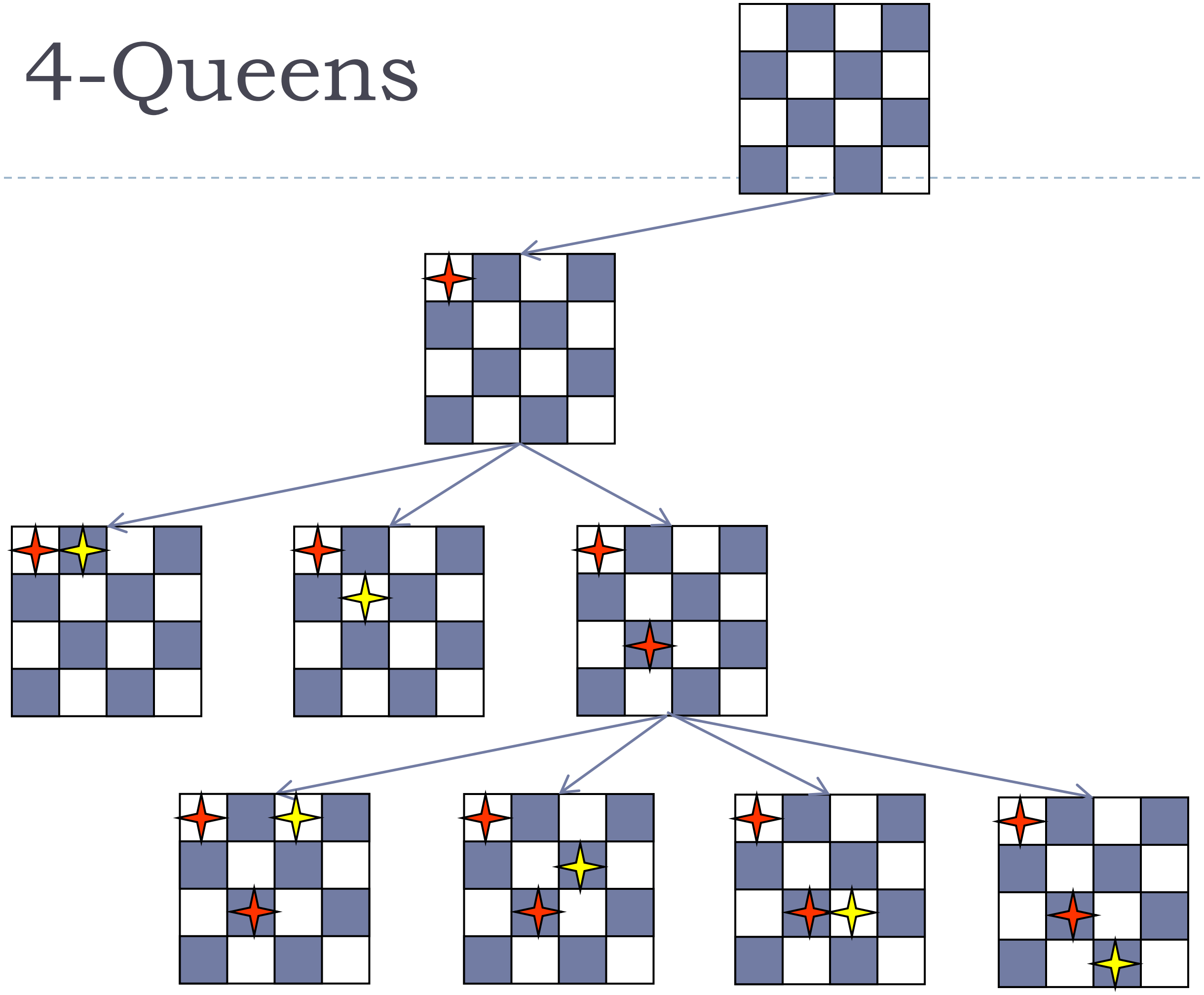
- Nodes are <u>partial assignments</u>
- <u>Incremental completion</u>
  - Each partial candidate is the parent of all candidates that differ from it by a single extension step.
- Traverses the search tree in <u>depth first order</u>.
- At each node *c*
  - If it cannot be completed to a valid solution, the whole sub-tree rooted at *c* is skipped (<u>not promising</u> branches are *pruned*).
  - Otherwise, the algorithm (1) checks whether *c* itself is a <u>valid solution</u>, returns it; and (2) <u>recursively enumerates all sub-trees</u> of *c*.

General backtracking search

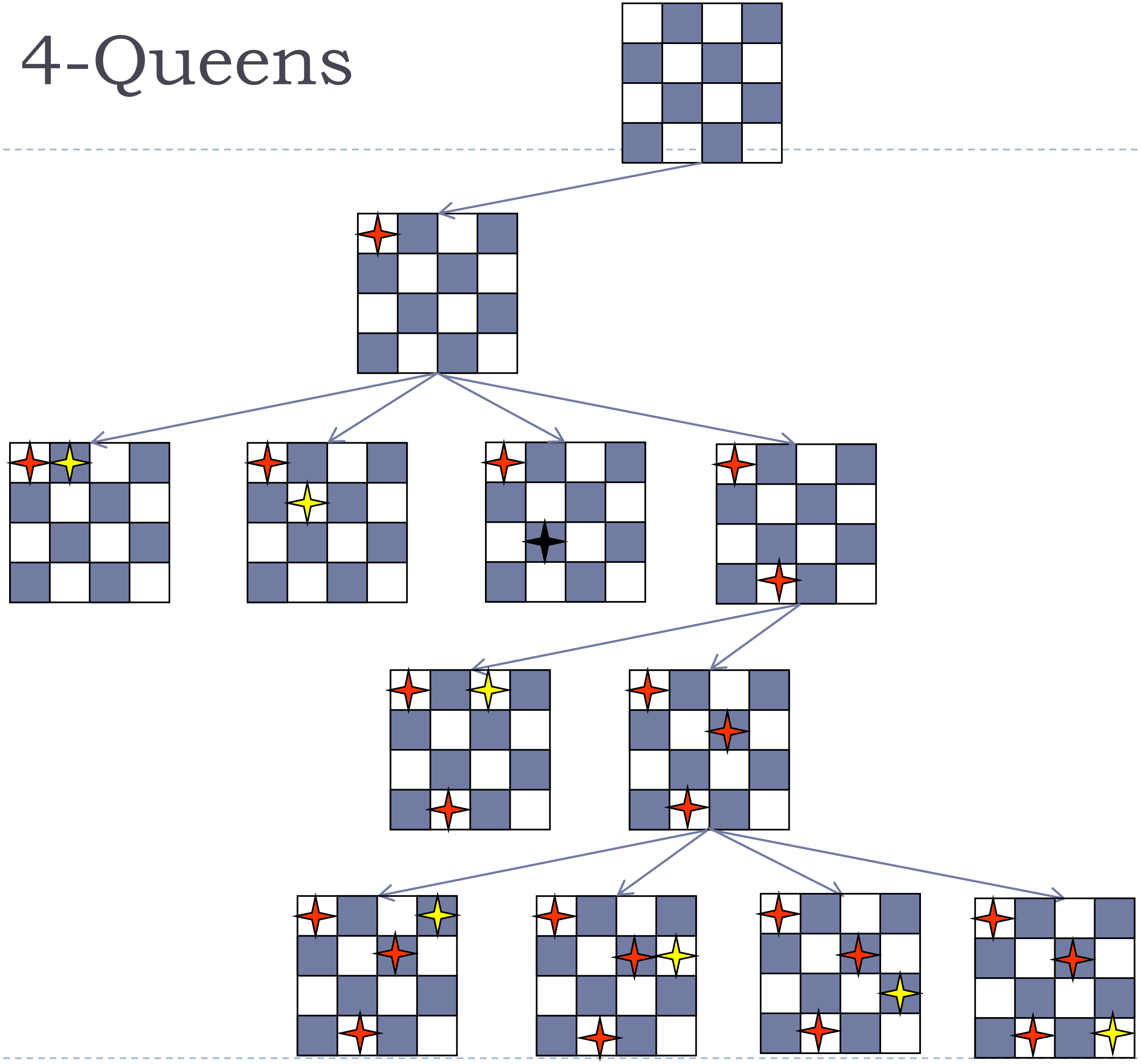
```
function BACKTRACK(v) returns a solution, or failure
    if there is a solution at v then return solution
    for each child u of v do
        if Promising(u) then
            result ← BACKTRACK(u)
            if result ≠ failure return result
    return failure
```

```
function BACKTRACK(assignment, csp) returns an assignment, or failure
    If assignment is complete then return assignment
    var ← select an unassigned variable
    for each val in Domain(var) do
        if Consistent(assignment ∪ {var ← value}, csp) then
            result ← BACKTRACK(assignment ∪ {var ← value}, csp)
            if result ≠ failure return result
    return failure
```

4-Queens



4-Queens



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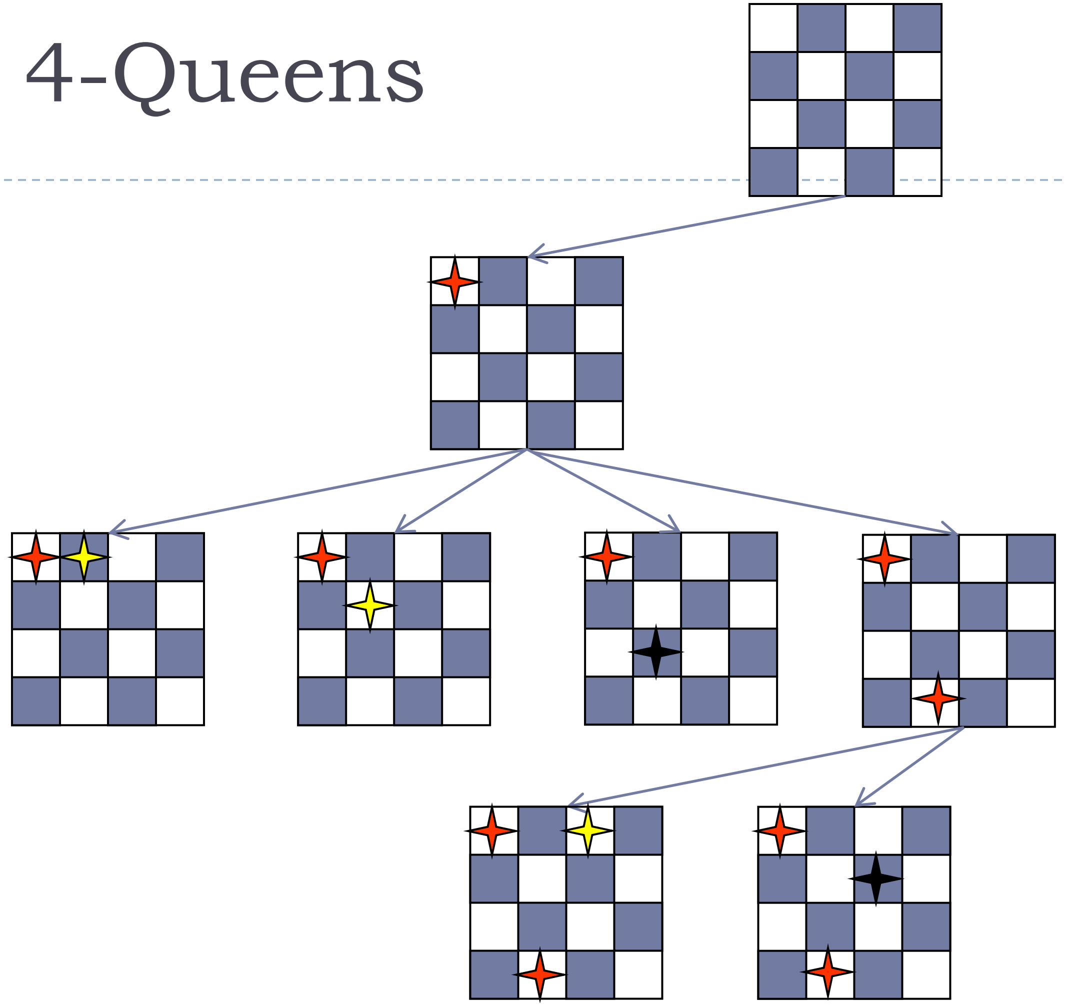

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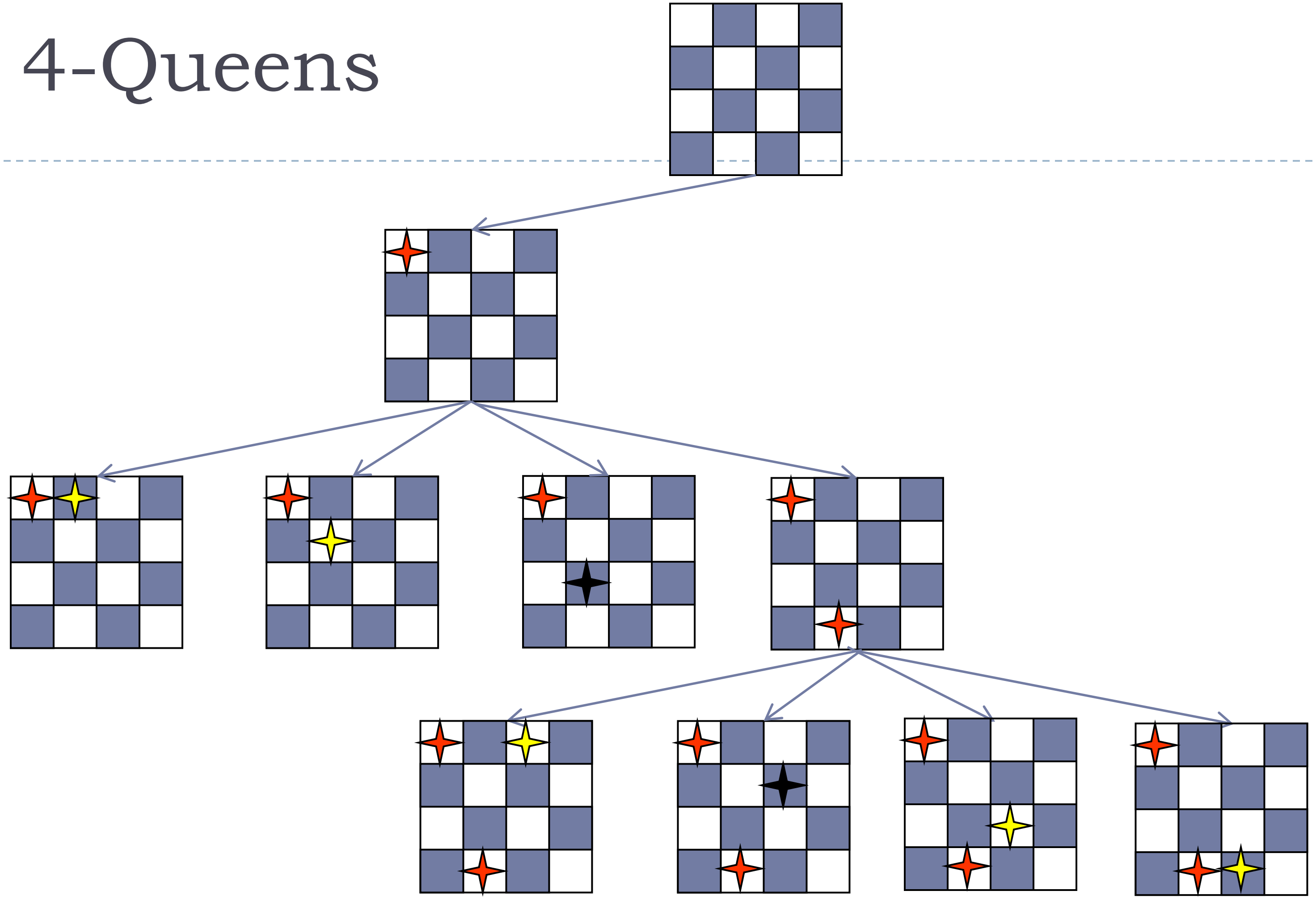

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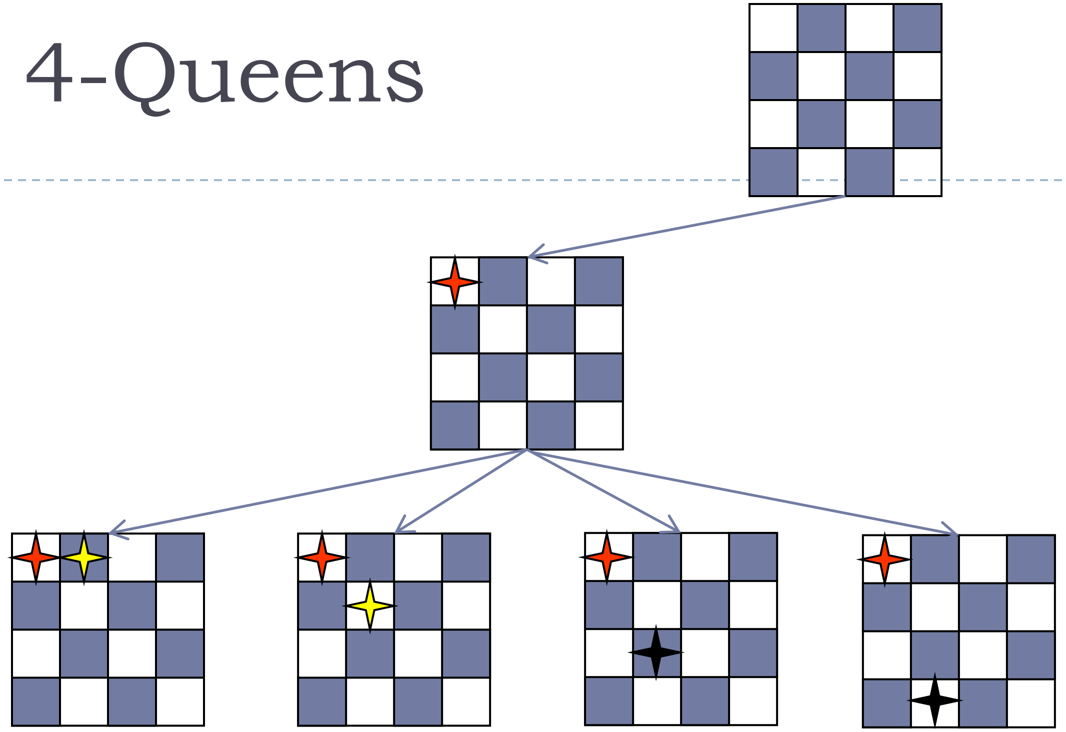

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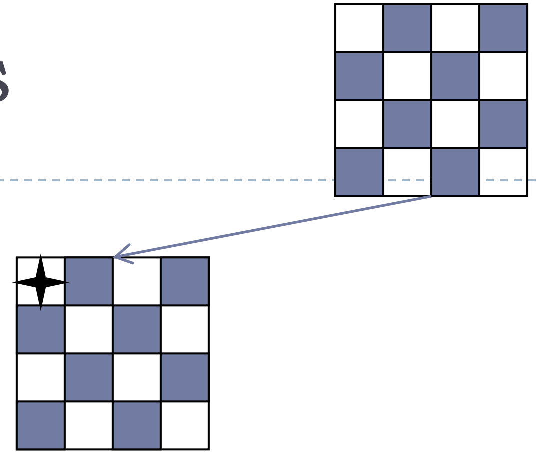

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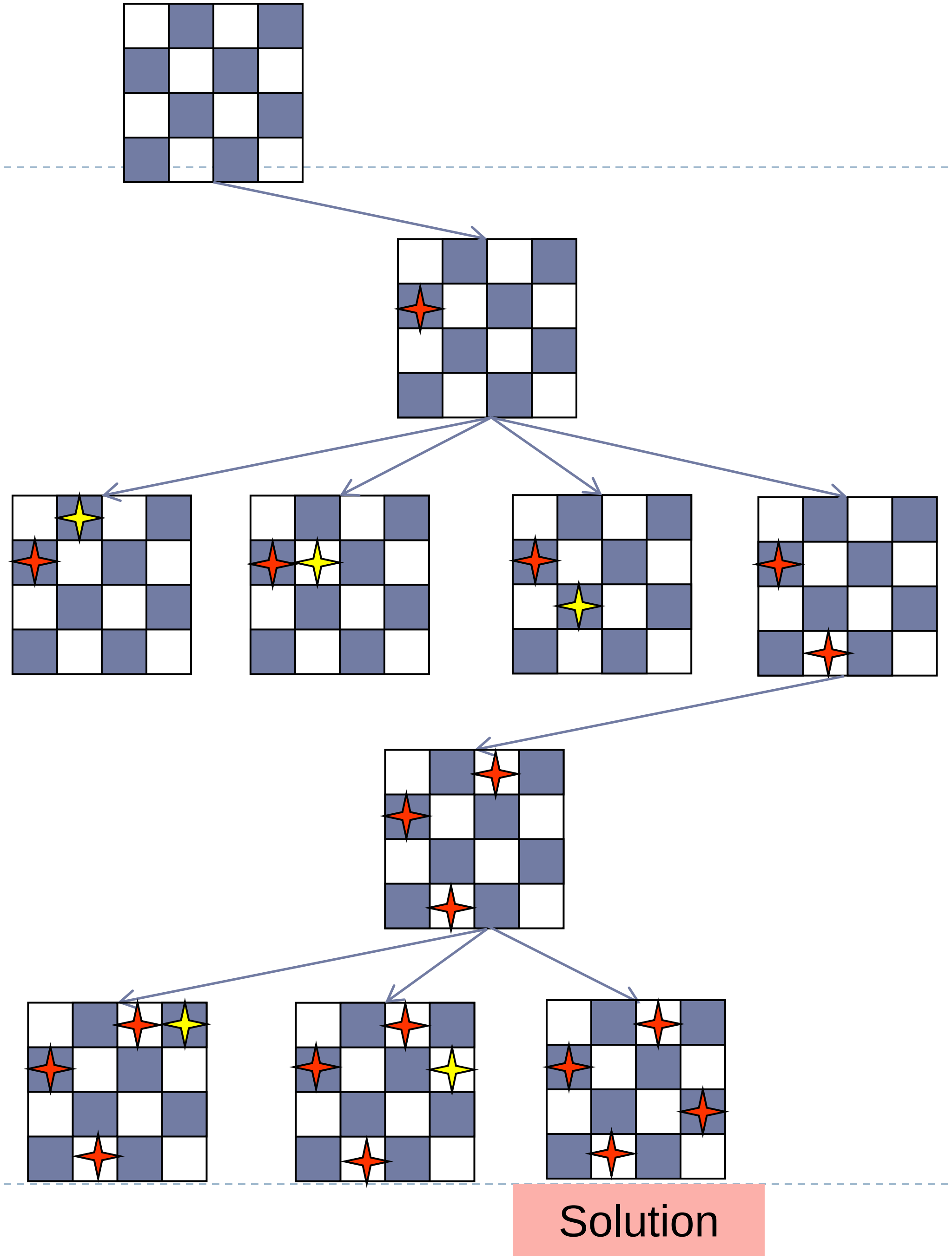
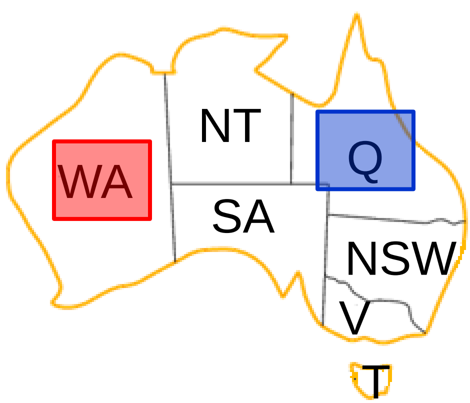
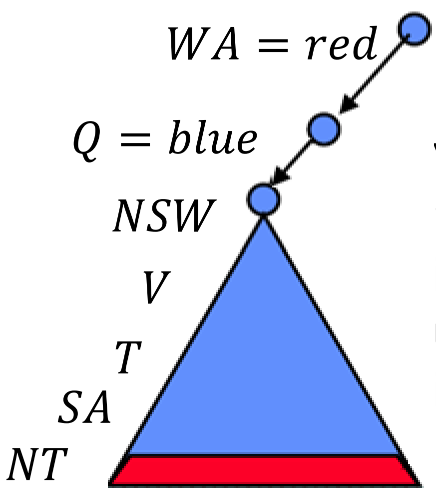


Naïve backtracking (late failure)

- Map coloring with three colors
  - $\{WA = red, Q = blue\}$ can not be completed.
  - However, the backtracking search does not detect this before selecting but NT and SA variables





Variable NT has no possible value. But it is not detected until tying to assign it a value.

Inference (looking ahead) in solving CSPs

- Inference as a <u>preprocessing stage</u>
  - AC-3 (arc consistency) algorithm
    - Removes values from domains of variables (and propagates constraints) to provide all constrained pairs of variables arc consistent.
- Inference <u>intertwined with search</u>
  - Forward checking
    - When selecting a value for a variable, infers new domain reductions on neighboring unassigned variables
  - Maintaining Arc Consistency (MAC) - Constraint propagation
    - Forward checking + recursively propagating constraints when changes are made to the domains
  - …

Local consistency

- Node consistency (1-consistency)
  - Unary constraints are satisfied.
- Arc consistency (2-consistency)
  - Binary constraints are satisfied.
- Path consistency (3-consistency)
- k- consistency
- Global constraints

Arc consistency

- (X_i, X_j) (X_i is <u>arc-consistent</u> with respect to X_j)
  <u>if for every value in D_i there is a consistent value in D_j</u>

- Example
  - Variables: $X = \{X_1, X_2\}$
  - Domain: $\{0,1,2, \dots, 9\}$
  - Constraint: $X_1 = {X_2}^2$

  - Is X_1 is arc-consistent w.r.t. X_2?
    - No, to be arc-consistent $Domain(X_1) = \{0,1,4,9\}$
  - Is X_2 is arc-consistent w.r.t. X_1?
    - No, to be arc-consistent $Domain(X_2) = \{0,1,2,3\}$

Arc consistency algorithm (AC-3)

function $AC\_3(csp)$ **returns** false if an inconsistency is found and true otherwise
  inputs: csp, a binary CSP with components X, D, C
  local variables: $queue$, a queue of arcs, initially all the arcs in csp

while $queue$ is not empty **do**
  $(Xi, Xj) \leftarrow REMOVE\_FIRST(queue)$
  if $REVISE(csp, X_i, X_j)$ **then**
    If size of $D_i = 0$ **then return** $false$
    for each X_k **in** $X_i.NEIGHBORS - \{X_j\}$ **do**
      add (X_k, X_i) to $queue$

function $REVISE(csp, X_i, X_j)$ **returns** true iff we revise the domain of X_i
  $revised \leftarrow false$
  for each x **in** D_i **do**
    if no value y in D_j allows (x, y) to satisfy the constraint between X_i and X_j **then**
      delete x from D_i
      $revised \leftarrow true$
return $revised$

Makes X_i arc-consistent with respect to X_j

AC-3

For each arc (X_i, X_j) in the queue

Remove it from queue

Makes X_i arc-consistent with respect to X_j

1) If D_i remains unchanged then continue
2) If $|D_i| = 0$ then return false
3) For each neighbor X_k of X_i except to X_j do

   add (X_k, X_i) to queue

If domain of X_i loses a value, neighbors of X_i must be rechecked

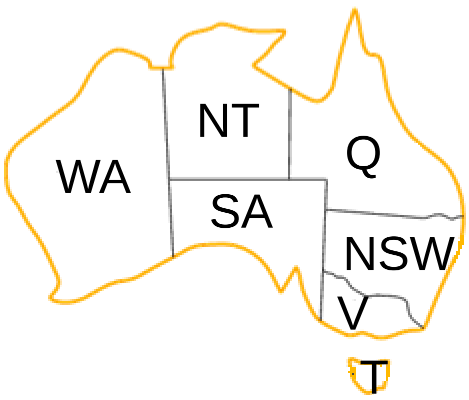
- Removing a value from a domain may cause further inconsistency, so we have to repeat the procedure until everything is consistent.
- When queue is empty, resulted CSP is equivalent to the original CSP.
  - Same solution (usually reduced domains speed up the search)

AC-3: time complexity

- Time complexity (n variables, c binary constraints, d domain size): $O(cd^3)$
  - Each arc (X_k, X_i) is inserted in the queue at most d times.
    - At most all values in domain X_i can be deleted.
  - Checking consistency of an arc: $O(d^2)$

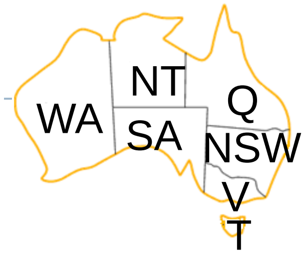
Arc consistency: map coloring example

- For general map coloring problem all pairs of variables are arc-consistent if $|D_i| \geq 2 (i = 1, \ldots, n)$



- Arc consistency can do nothing.
  - Fails to make enough inference
- We need stronger notion of consistency to detect failure at start.
  - 3-consistency (path consistency): for any consistent assignment to each set of two variables, a consistent value can be assigned to any other variable.
  - Both of the possible assignments to set $\{WA, SA\}$ are inconsistent with NT.

k-consistency



- Arc consistency does not detect all inconsistencies
  - Partial assignment $\{WA = red, NSW = red\}$ is inconsistent.

- A CSP is k-consistent if for any set of $k-1$ variables and for any consistent assignment to those variables, a consistent value can always be assigned to any kth variable.
  - E.g. 1-consistency = node-consistency
  - E.g. 2-consistency = arc-consistency
  - E.g. 3-consistency = path-consistency

- Strongly k-consistent:
  - k-consistent for all values {k, k-1, …2, 1}

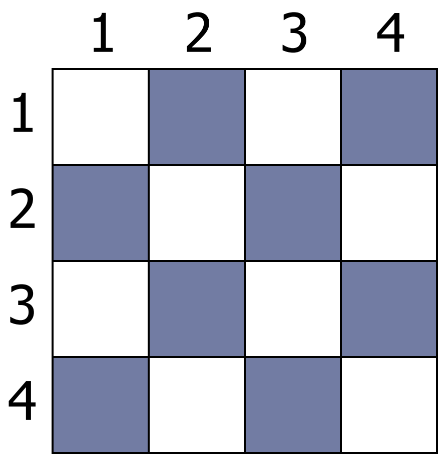
Which level of consistency?

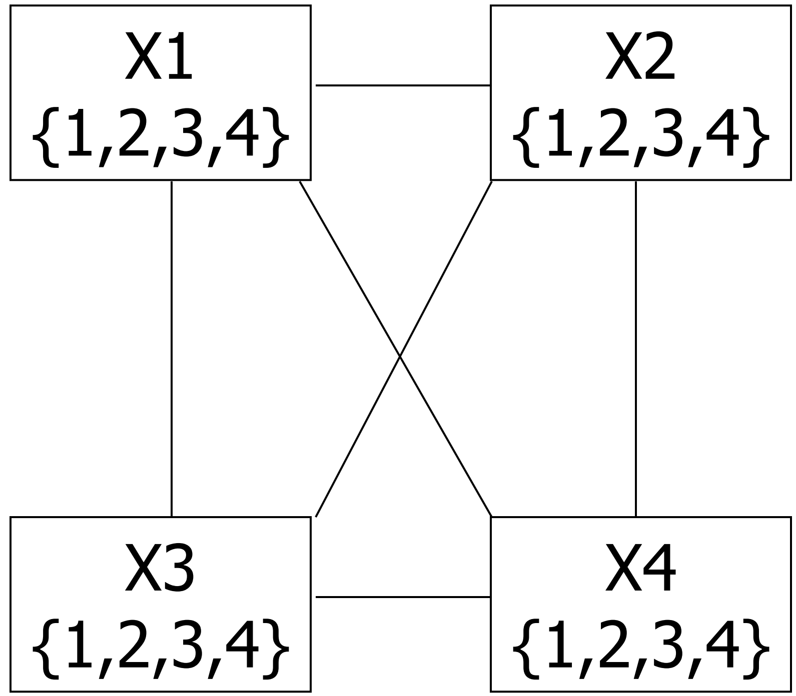
- Trade off between the required time to establish k-consistency and amount of the eliminated search space.
  - If establishing consistency is slow, this can slow the search down to the point where no propagation is better.
- Establishing k-consistency need exponential time and space in k (in the worst case)
- Commonly computing 2-consistency and less commonly 3-consistency

Inference during the search process

- It can be more powerful than inference in the preprocessing stage.

Example: 4-Queens





4-Qeens slides have been adopted from Dorr's slides on CMSC-421 course

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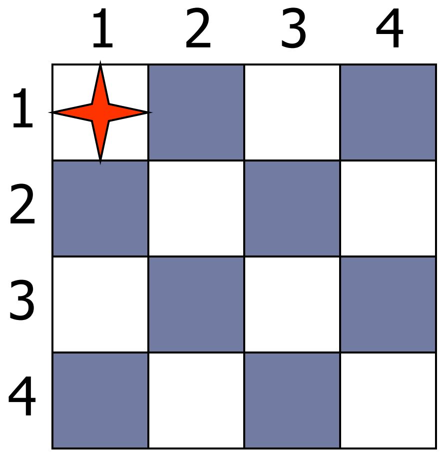

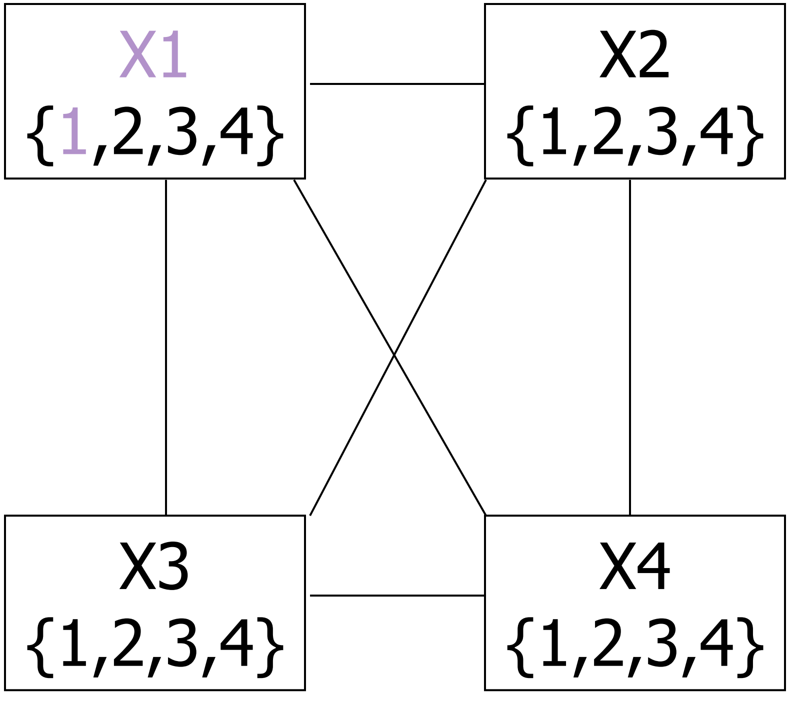

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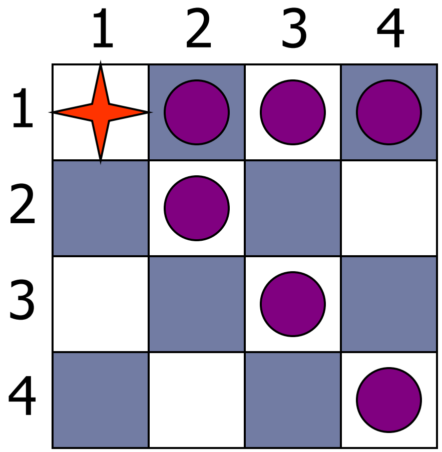

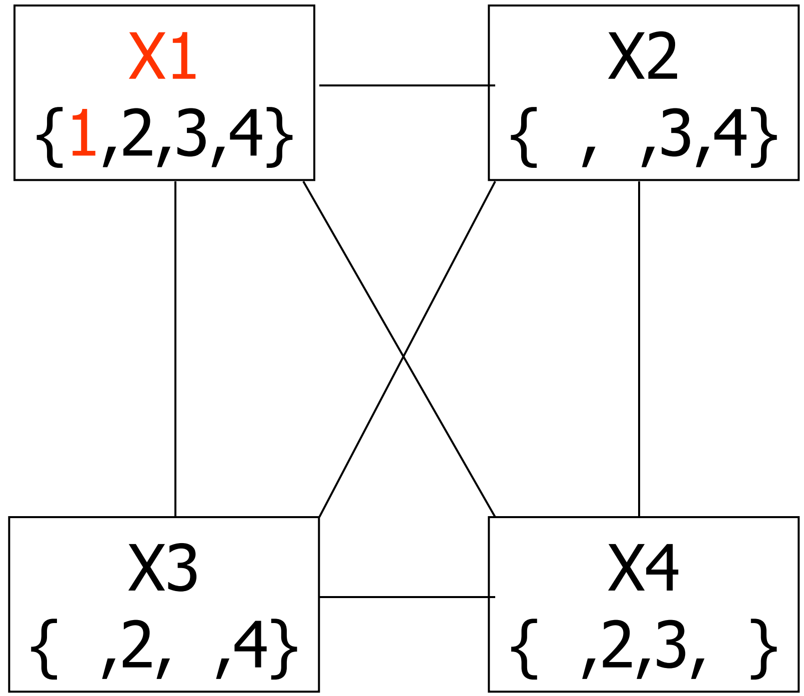

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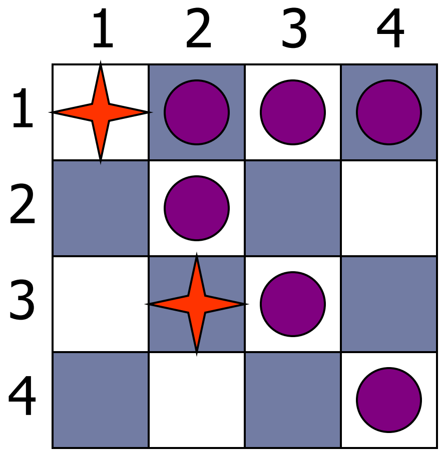

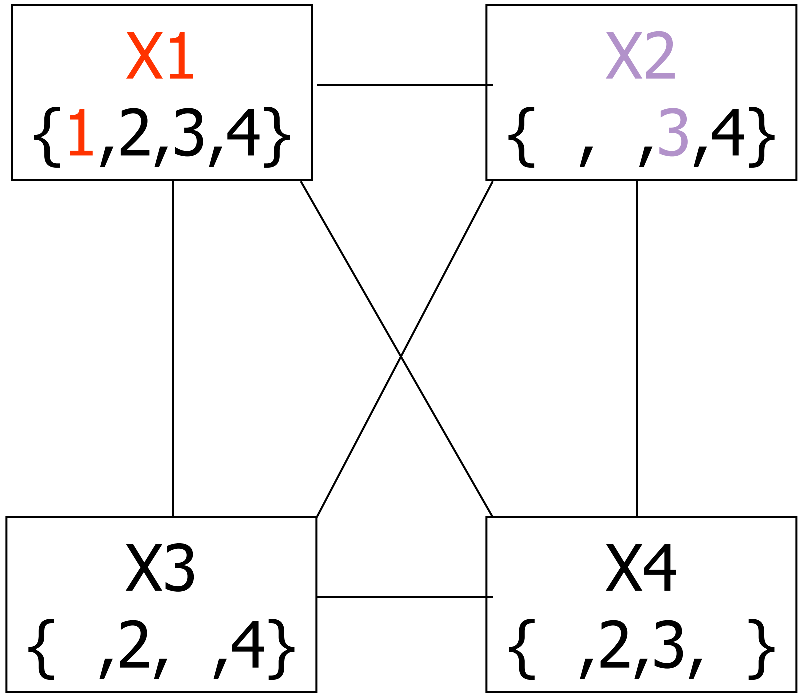

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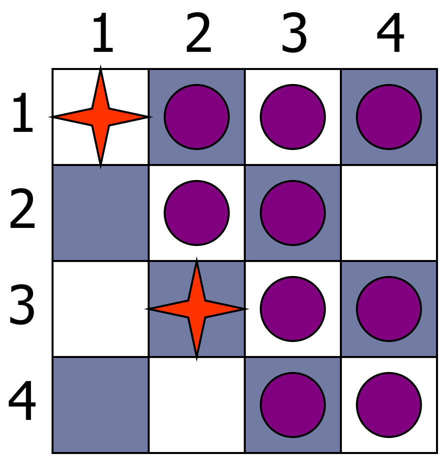

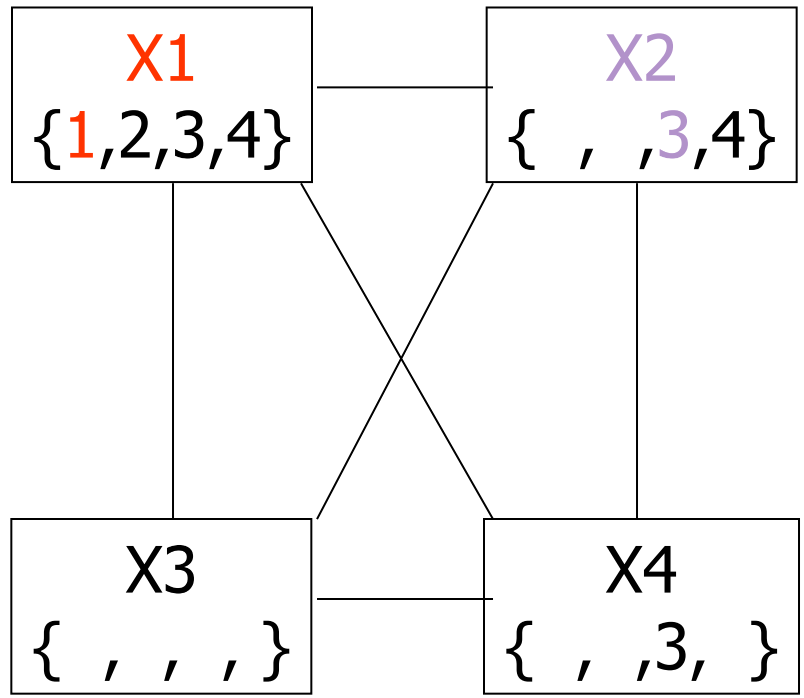

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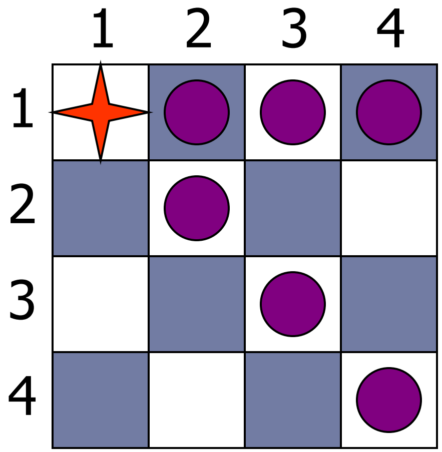
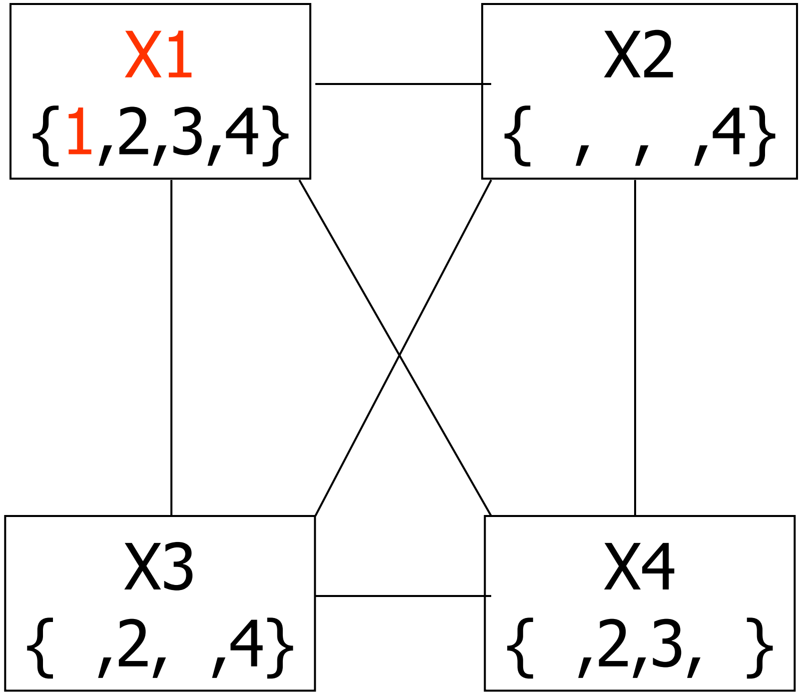
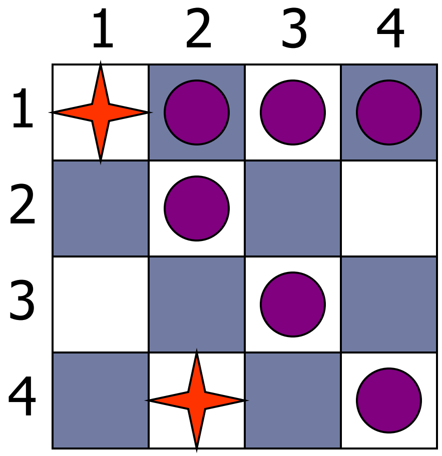
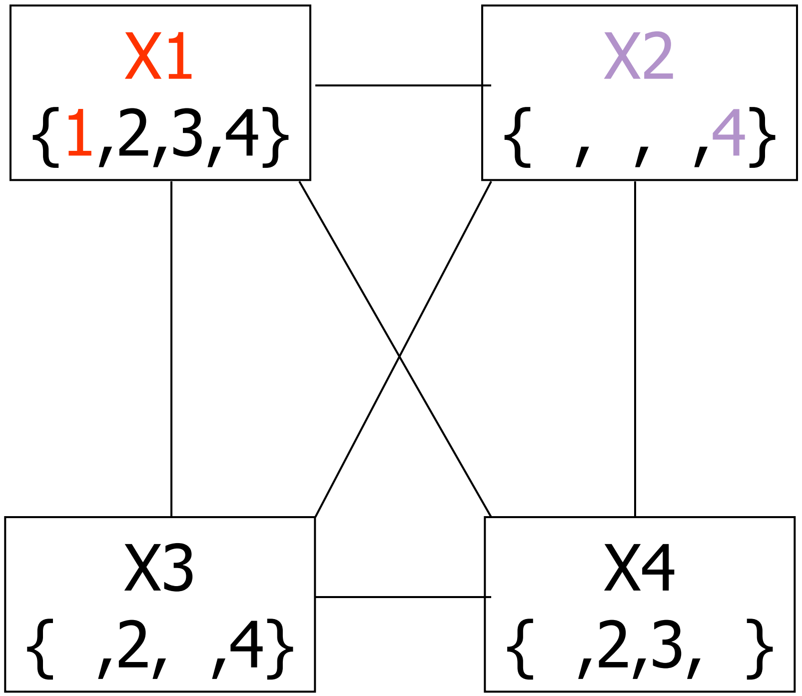


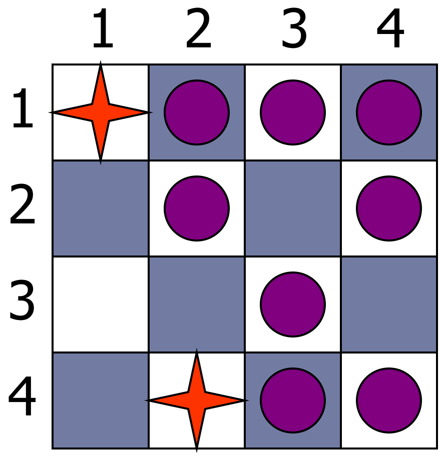


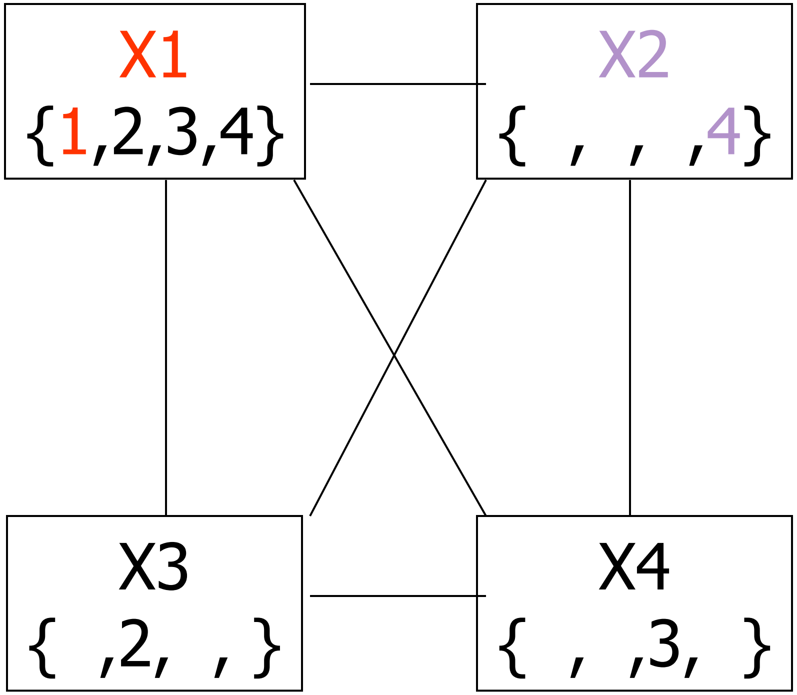
Example: 4-Queens



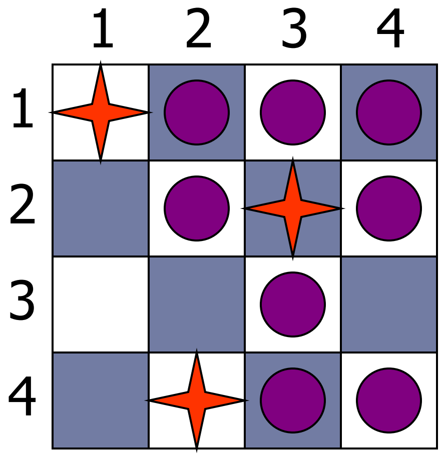


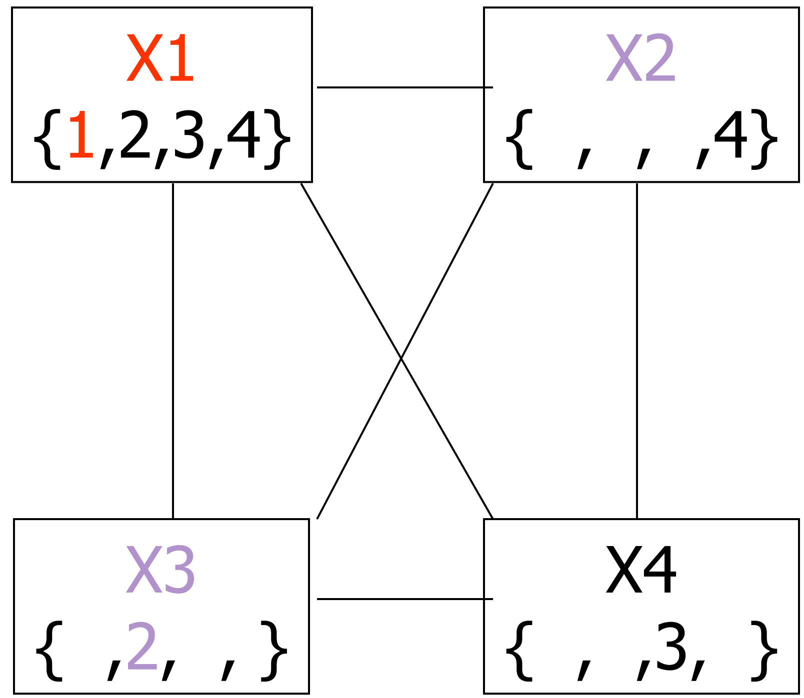
Example: 4-Queens



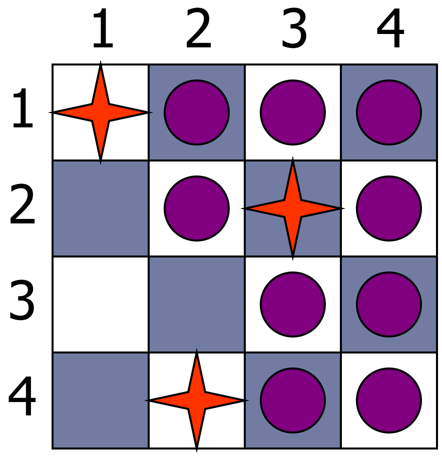


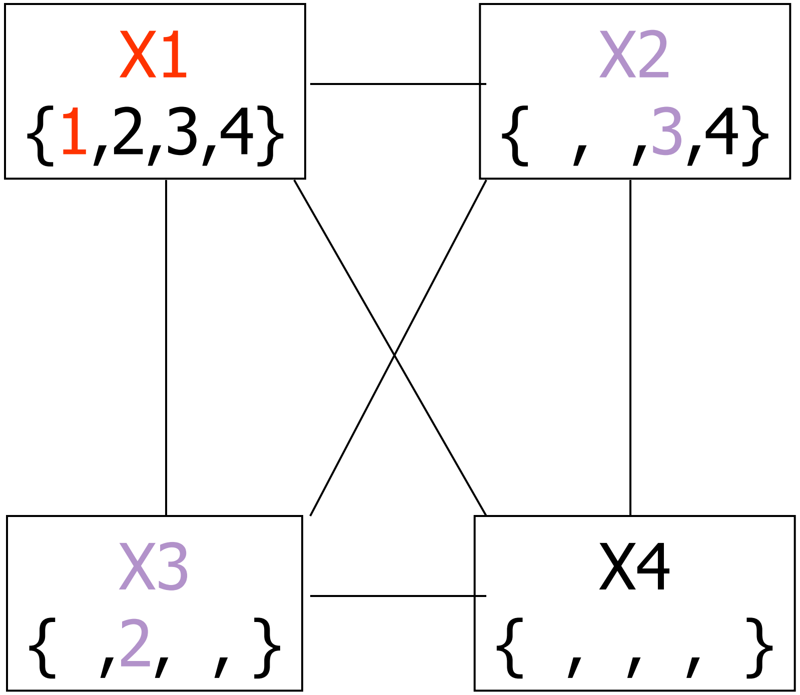
Example: 4-Queens





Example: 4-Queens





CSP backtracking search

```
function BACKTRACKIN_SEARCH(csp) returns a solution, or failure
    return BACKTRACK({ }, csp)
```

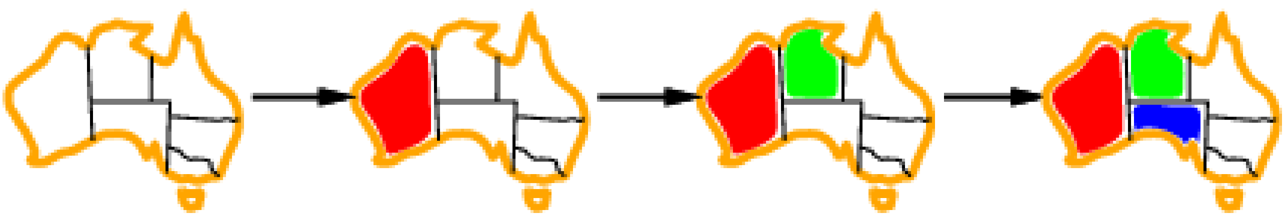
```
function BACKTRACK(assignment, csp) returns a solution, or failure
    if assignment is complete then return assignment
    var ← SELECT_UNASSIGNED_VARIABLE(csp)
    for each value in ORDER_DOMAIN_VALUES(var, assignment, csp) do
        if value is consistent with assignment then
            add {var = value} to assignment
            inferences ← INFERENCE(csp, var, value)
            if inferences ≠ failure then
                add inferences to assignment
                result ← BACKTRACK(assignemnt, csp)
                if result ≠ failure then return result
        remove {var = value} and inferences from assignment
return failure
```

Solving CSP efficiently

- Which variable should be assigned next?
  - *SELECT_UNASSIGNED_VARIABLE*
- In what order should values of the selected variable be tried?
  - *ORDER_DOMAIN_VALUES*
- What inferences should be performed at each step in the search?
  - *INFERENCE*
- When the search arrives at an assignment that violates a constraint, can the search avoid repeating this failure?

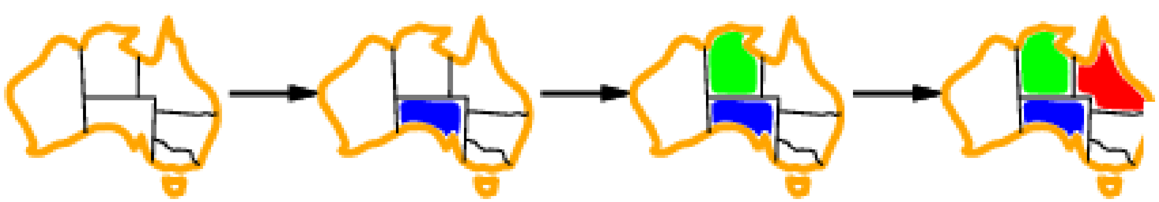
Minimum Remaining Values (MRV)

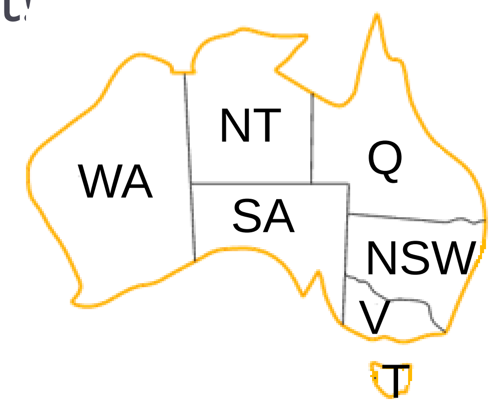
- Chooses the variable with the fewest legal values
  - Fail first
- Also known as Most Constrained Variable (MCS)
- Most likely to cause a failure soon and so pruning the search tree

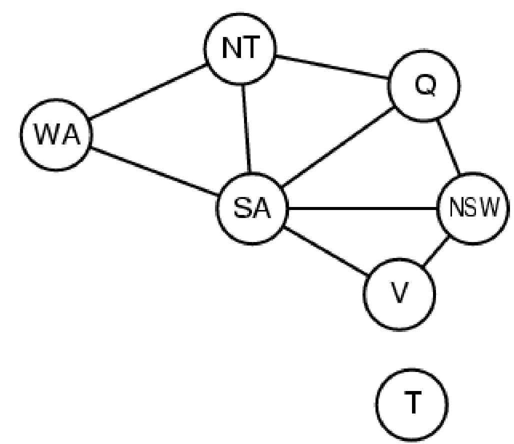


Degree heuristic

- Tie-breaker among MRV variables
- Degree heuristic: choose the variable with the most constraints on remaining variables
  - To choose one who interferes the others most!
  - reduction in branching factor

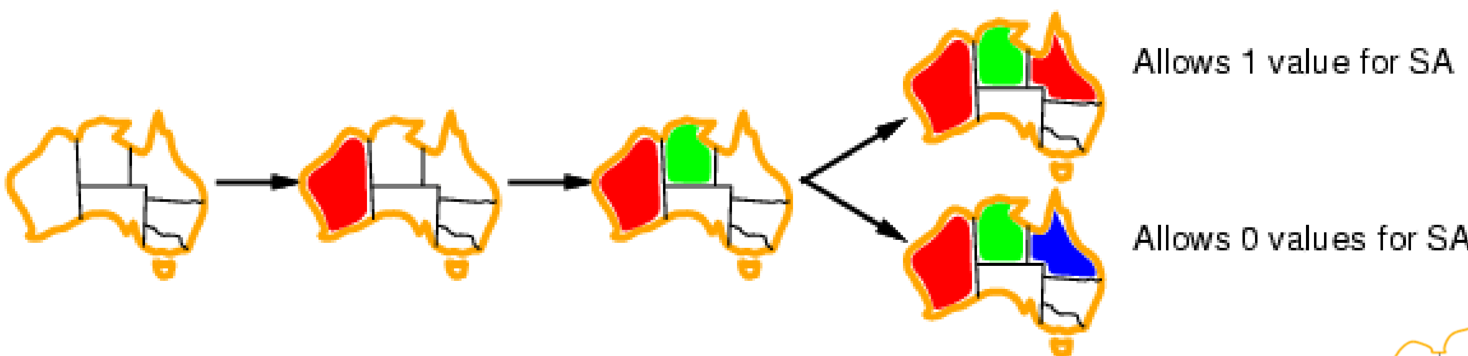




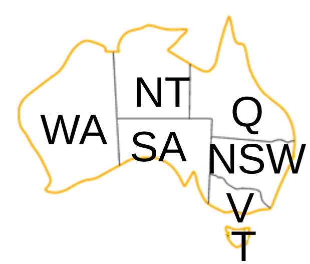


Least constraining value

- Given a variable, choose the least constraining value:
  - one that rules out the fewest values in the remaining variables
  - leaving maximum flexibility for subsequent variable assignments
    - Fail last (the most likely values first)

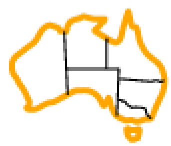


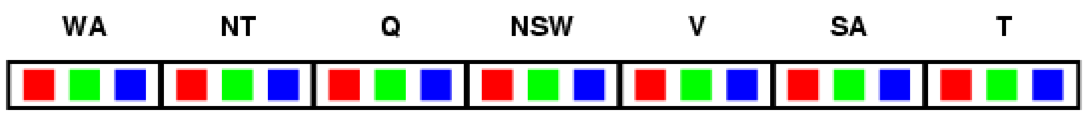
- Assumption: we only need one solution

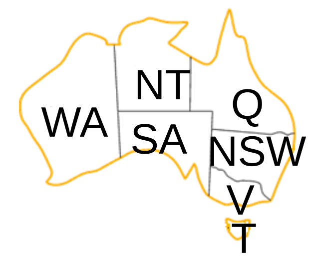


Forward Checking (FC)

- When selecting a value for a variable, infer <u>new domain reductions on neighboring unassigned variables</u>.
  - Terminate search when a variable has no legal value
- When X is assigned, FC establishes arc-consistency for it.







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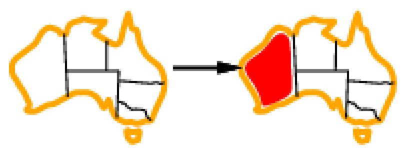
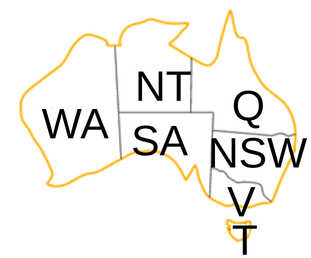
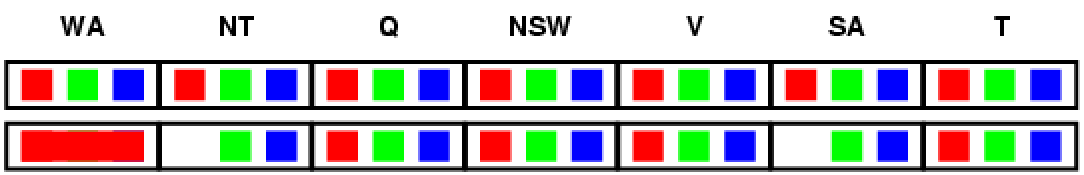




$WA = red$



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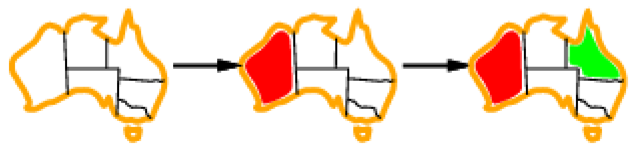

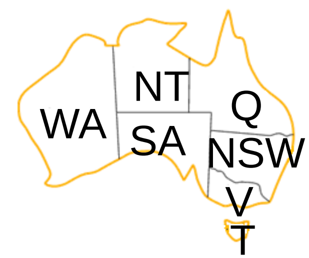

$WA = red$

$Q = green$

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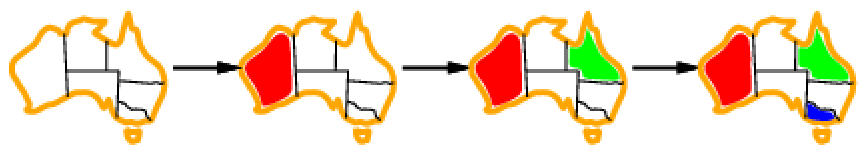
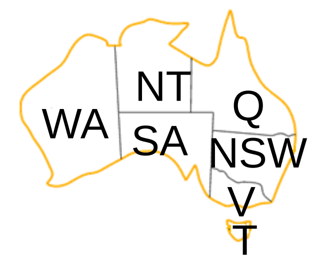




$WA = red$

$Q = green$

$V = blue$

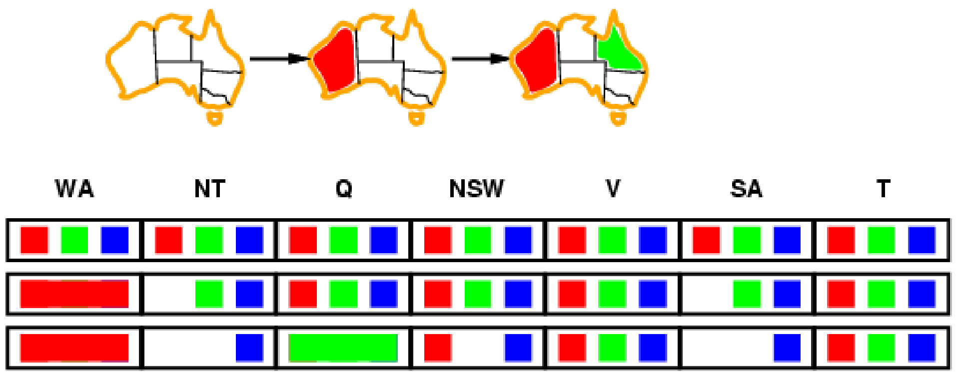
$\Longrightarrow \{WA = red, Q = green, V = blue\}$ is an inconsistent partial assignment

Forward Checking (FC)

- No need to FC if arc consistency (AC-3) have been done as a preprocessing stage.
- Combination of MRV heuristic and FC is more effective.
  - FC incrementally computes the information that MRV needs.

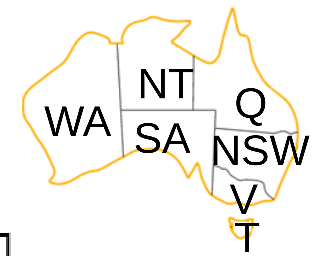
Constraint propagation

- FC makes the current variable arc-consistent but does not make all the other variables arc-consistent



$WA = red$

$Q = green$



- NT and SA cannot both be blue!
  - FC does not look far enough ahead to find this inconsistency
- Maintaining Arc Consistency (MAC) - Constraint propagation
  - Forward checking + recursively propagating constraints when changing domains (similar to AC-3 but only arcs related to the current variable are put in the queue at start)

Solving CSPs by local search algorithms

- In the CSP formulation as a search problem, path is irrelevant, so we can use complete-state formulation

- State: an assignment of values to variables
- Successors(s): all states resulted from s by choosing a new value for a variable
- Cost function h(s): Number of violated constraints
- Global minimum: $h(s) = 0$

function $MIN\_CONFLICTS(csp, \max\_steps)$ **returns** a solution or failure

  inputs: csp, a constraint satisfaction problem

    max_steps, the number of steps allowed before giving up

  $current \leftarrow$ an initial complete assignment for csp

  for $i = 1$ to $\max\_steps$ do

    if $current$ is a solution for csp **then return** current

    $var \leftarrow$ a randomly chosen conflicted variable from $csp.VARIABLES$

    $value \leftarrow$ the value v for var that minimizes $CONFLICTS(var, v, current, csp)$

    set $var = value$ in $current$

  return $failure$

if current state is consistent then
  return it
else
  choose a random variable v, and change assignment of v
  to a value that causes minimum conflict.

Local search for CSPs

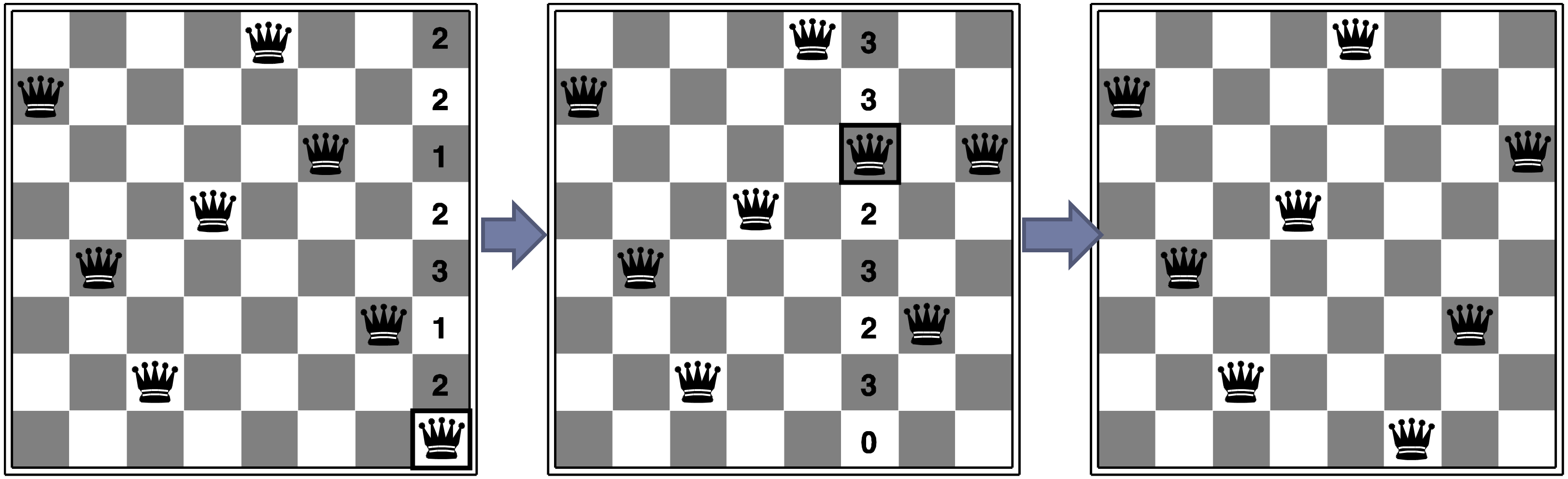
- Variable selection: randomly select any conflicted variable

- Value selection by min-conflicts heuristic
  - choose value that violates the fewest constraints
    - i.e., hill-climbing

- Given random initial state, it can solve *n*-queens in almost constant time for arbitrary *n* with high probability
  - $n = 1000000$ in an average of 50 steps

- N-queens is easy for local search methods (while quite tricky for backtracking)
  - Solutions are very densely distributed in the space and any initial assignment is guaranteed to have a solution nearby.

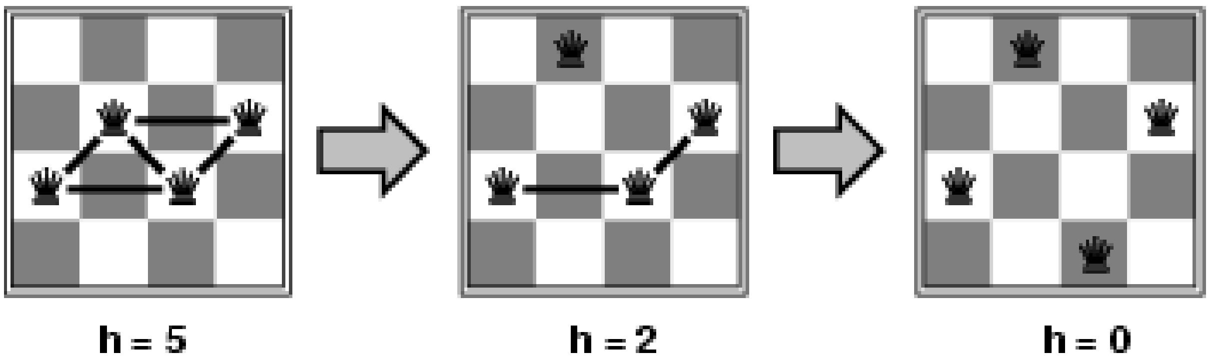
8-Queens example



$Q_8 = 3$

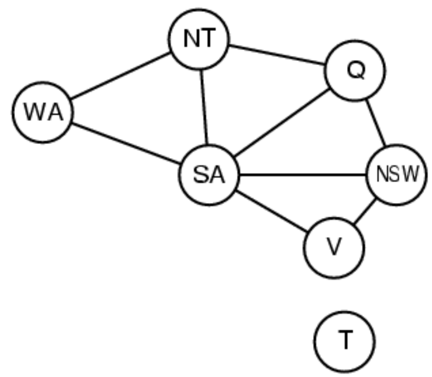
$Q_6 = 8$

4-Queens example



Graph Structure

- Connected components as independent sub-problems
  - The color of T is independent of those of other regions

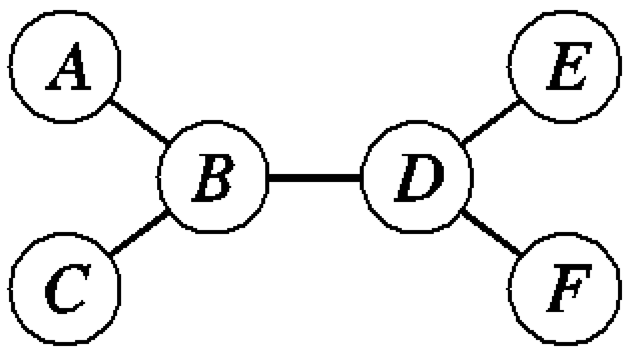


- Suppose each sub-problem has h variables out of n
  - Worst-case solution cost is $O((n/h)(d^h))$ that is linear in n

- Example: $n = 80, d = 2, h = 20$ (processing: 10^6 nodes/sec)
  - 40 billion years
  - 4 seconds

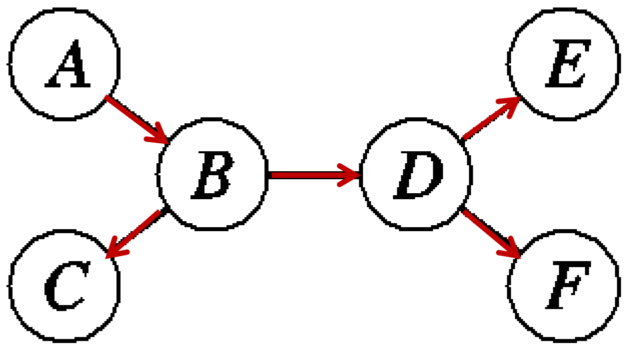
Tree structured CSPs

- Any two variables are connected by only one path
- Any tree-structured CSP can be solved in time <u>linear in n</u>

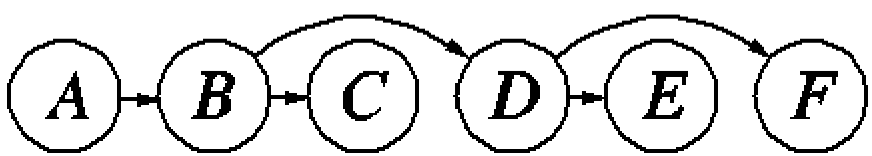


Tree structured CSPs: topological ordering

- Construct a rooted tree (picking any variable to be root, …)



- Order variables from root to leaves such that every node's parent precedes it in the ordering (<u>topological ordering</u>)



function $TREE\_CSP\_SOLVER(csp)$ **returns** a solution or failure

input: csp, a CSP with components X, D, C

$n \leftarrow$ number of variables in X
$assignment \leftarrow$ an empty assignment
$root \leftarrow$ any variable in X
$X \leftarrow TOPOLOGICAL(X, root)$
for $j = n$ **down to** 2 **do**
  $MAKE\_ARC\_CONSISTENT(PARENT(X_j), X_j))$
  if it cannot be made consistent **then return** $failure$
for $i = 1$ **to** n **do**
  $assignment[X_i] \leftarrow anyconsistent\ value\ from\ D_i$
  if there is no consistent value **then return** $failure$
return $assignment$

Why doesn't this algorithm work with loops?

Tree structured CSP Solver

$X \leftarrow$ Topological Sort
for $i = n$ downto 2 do
    Make-Arc-Consistent(Parent(X_i), X_i)
for $i = 1$ to n do
    $X_i \leftarrow$ any remaining value in D_i

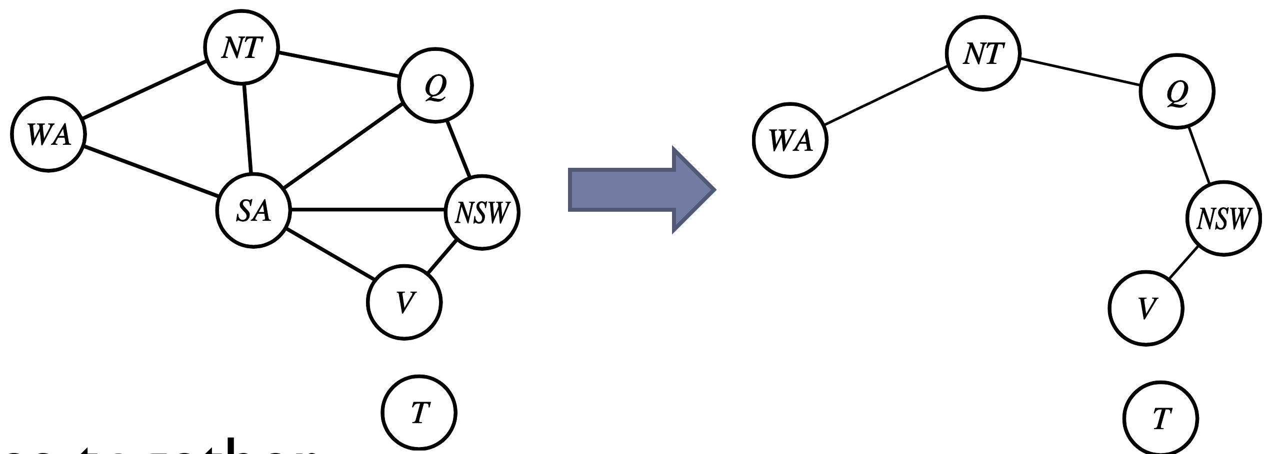
remove all values from domain of Parent(X_i) which may violate arc-consistency.

- After running loop1, any arc from a parent to its child is arc-consistent.

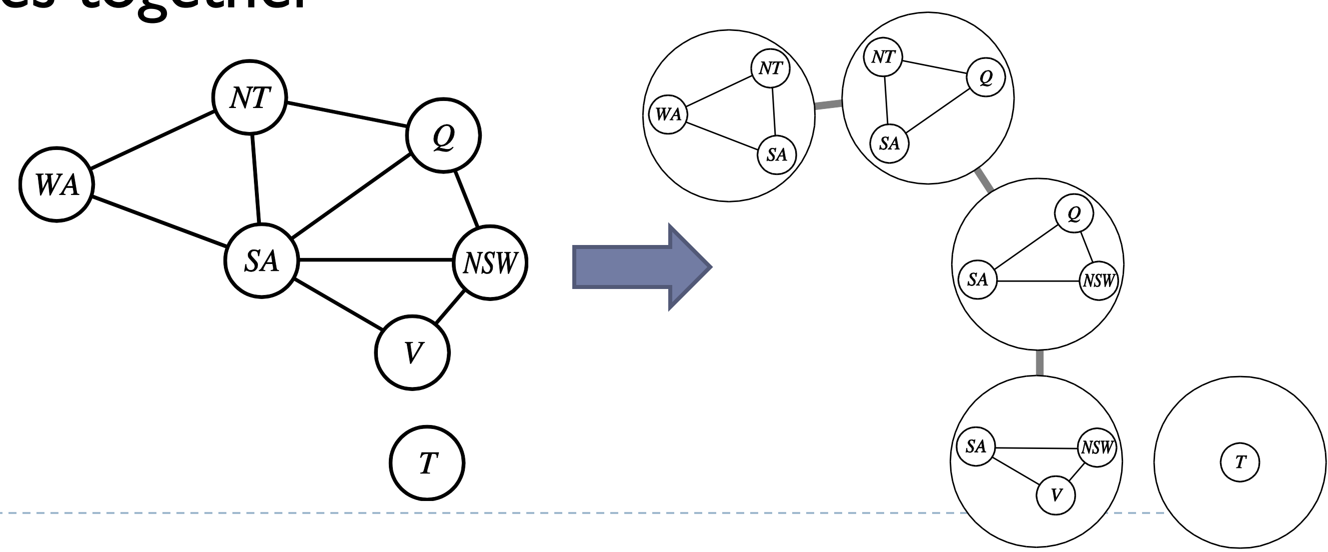
- $\Rightarrow$ if the constraint graph has no loops, the CSP can be solved in $O(nd^2)$ time.

Reduction of general graphs into trees

- Removing nodes



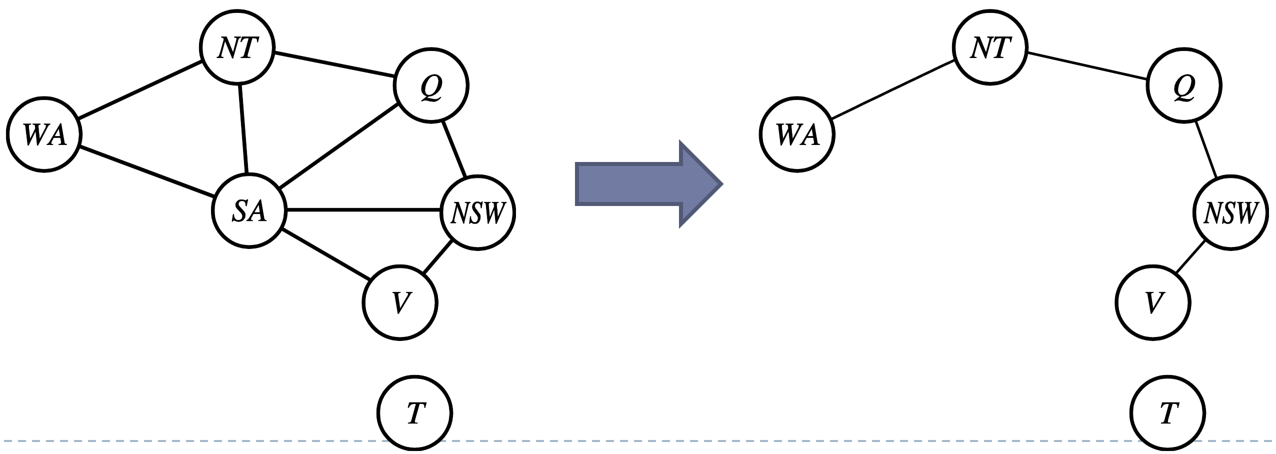
- Collapsing nodes together



Cut-set conditioning

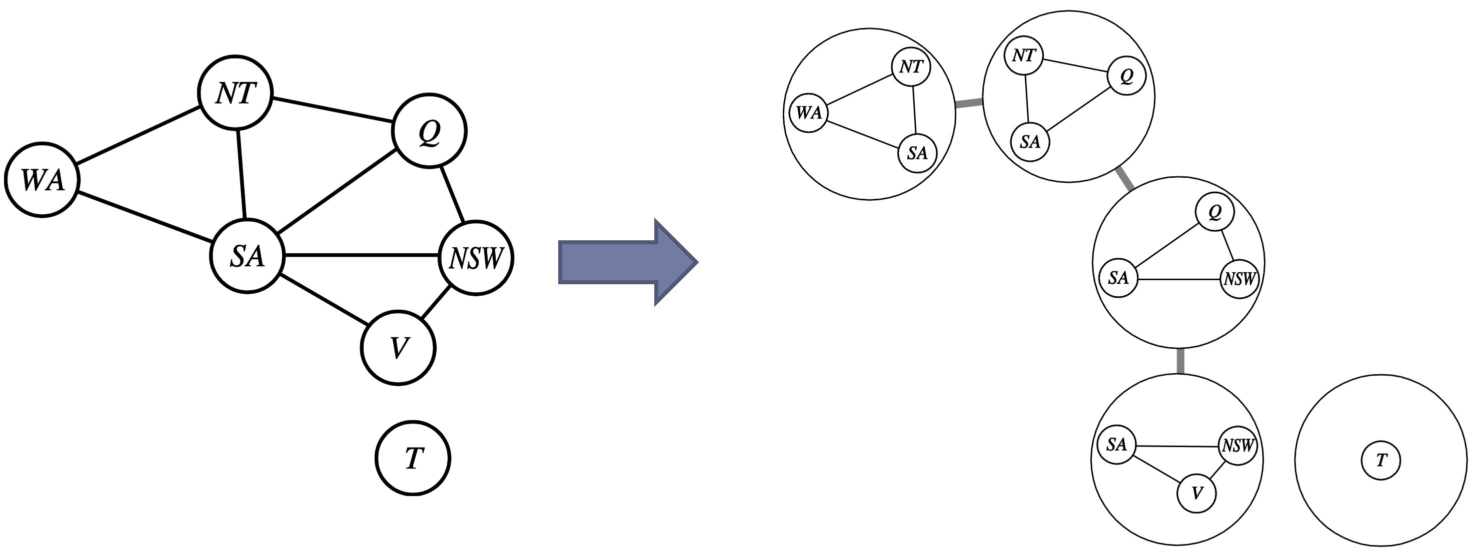
1) Find a <u>subset S</u> such that the remaining graph becomes a tree
2) For each possible consistent assignment to S
    a) remove inconsistent values from domains of remaining variables
    b) solve the remaining CSP which has a tree structure

- **Cutset size c gives runtime $O((d^c)\ (n-c)\ d^2\ )$**
  - very fast for small c
  - c can be as large as $n-2$



Tree Decomposition

- Create a tree-structured graph of overlapping sub-problems (each sub-problem as a mega-variable)
- Solve each sub-problem (enforcing local constraints)
- Solve the tree-structured CSP over mega-variables



Summary

- CSP benefits
  - Standard representation of many problems
  - Generic heuristics (no domain specific expertise)

- CSPs solvers (based on classical search)
  - Backtracking
  - Variable ordering and value selection heuristics
  - Forward checking prevents one-step more assignments that guarantee later failure.
  - Constraint propagation: in addition to forward checking propagates constraints (to detect some inconsistencies earlier)

- Iterative min-conflicts (based on local search) is usually effective in solving CSPs.

- Graph structure may be useful in solving CSPs efficiently.