Homework 4 (Chapter 4)

Problems

1. Derive the following formulas in your own word. You will get a zero for the whole homework if you copy the text from your book. Explain each line of the proof.
   a. \( x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} X(j\omega)e^{-j\omega t} d\omega \)
   b. \( X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t} dt \)

2. Suppose that real signal \( x(t) \) has the Fourier Transform \( X(j\omega) \). Suppose another signal \( y(t) \) has the same shape as \( X(j\omega) \).
   a. Determine \( Y(j\omega) \) Fourier Transform in terms of \( X(j\omega) \).
   b. Using the result of the last part show that:
      \[ F\{e^{-jAt}\} = 2\pi\delta(\omega + A) \]

3. Determine the continuous-time signal corresponding to each of the following transforms:
   a. \( X(j\omega) = j[\delta(\omega + 2) + \delta(\omega - 1) * j] - [\delta(\omega + 1) + \delta(\omega - 2) * j] \)
   b. \( X(j\omega) = 2\cos(4\omega - \pi/2) \)

4. Compute the Fourier transform of each of the following signals:
   a. \( [e^{-\alpha t} \cos 2\omega_0 t] u(t), \alpha > 0 \)
   b. \( x(t) = e^{-|2t|} \sin 2t \)
   c. \( x(t) = \begin{cases} 1 + \sin \pi t, & |t| \leq 2 \\ 0, & |t| > 2 \end{cases} \)
   d. \( \frac{\sin \pi t}{\pi(t-1)}[\sin \pi(t-1)]_t \)
   e. \( x(t) = \begin{cases} 2 + t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases} \)
   g. The signal \( x(t) \) depicted below:

   ![Signal Diagram](image-url)
5. Determine which, if any, of the real signals depicted in below have Fourier transforms that satisfy each of the following conditions:

a. \( \Re \{X(j\omega)\} + \Im \{X(j\omega)\} = 0 \)

b. There exists a real \( \alpha \) such that \( e^{j\alpha \omega}X(j\omega) \) is real

c. \( X(j0) = 0 \)

d. \( \int_{-\infty}^{\infty} X(j\omega)d\omega = 0 \)

e. \( \int_{-\infty}^{\infty} \omega X(j\omega)d\omega = 0 \)

f. \( X(j\omega) \) is not periodic

6. Let \( X(j\omega) \) denote the Fourier transform of the signal \( x(t) \) depicted in Figure P4.25 of textbook p. 341. (P 4.25 p. 341 with little change) **Note:** You should perform all these calculations without explicitly evaluating \( X(j\omega) \).

a. \( X(j\omega) \) can be written as \( A(j\omega)e^{j\theta(j\omega)} \) where \( A(j\omega) \) and \( \theta(h\omega) \) are real. Find \( \theta(j\omega) \).

b. Find \( e^{-j}X(j0) \).

c. Find \( \int_{-\infty}^{\infty} X(j\omega)d\omega \).

d. Evaluate \( \int_{-\infty}^{\infty} |X(j\omega)|^2d\omega \).
e. Sketch the inverse Fourier transform of $\mathcal{R}e\{X(j\omega)\}$ and $\mathcal{I}m\{X(j\omega)\}$.

7. The output of a casual LTI system is related to the input $x(t)$ by the equation:

$$\frac{dy}{dt} + \frac{d^2y}{dt^2} = \int_{-\infty}^{\infty} x(\tau)z(t - \tau)d\tau + x(t)$$

where $z(t) = e^{-2t}u(t) + \delta(t)$
determin the impulse response of the system.

8. Let $g_1(t) = \{[\cos(\omega_0t)]x(t)\} * h(t)$ and $g_2(t) = \{[\sin(\omega_0t)]x(t)\} * h(t)$ where

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk100t}$$

is a real-valued periodic signal that $h(t)$ is the impulse response of a stable LTI system.

a. Specify a value for $\omega_0$ and any necessary constraints on $H(j\omega)$ to ensure that $g_1(t) = \mathcal{R}e\{a_5\}$ and $g_2(t) = \mathcal{I}m\{a_5\}$.

b. Give an example of $h(t)$ such that $H(j\omega)$ satisfies the constraints you specified in part (a). (P4.42 p. 348)

**Practical Assignment**

1. Consider a discrete-time system $H_1$ with impulse response

$$h_1[n] = u[n] - u[n - 1] + u[n - 2] - u[n - 5],$$

a discrete-time system $H_2$ with impulse response

$$h_2[n] = \left(\frac{1}{3}\right)^n (u[n + 4] - u[n - 4]),$$

and a discrete-time signal

$$x[n] = \left(\frac{1}{2}\right)^n (u[n - 1] - u[n - 5]).$$

The signals $h_1[n]$, $h_2[n]$, and $x[n]$ are all defined for $-10 \leq n \leq 10$.

a. Plot $h_1[n]$, $h_2[n]$, and $x[n]$ together using the `subplot` function.

b. Consider a system $H$ formed from the series connection of $H_1$ and $H_2$, where $x[n]$ is input to $H_1$, the output $v[n]$ of $H_1$ is input to $H_2$, and the output of $H_2$ is $y[n]$. Use the `conv` function to find $v[n]$ and $y[n]$. Plot $v[n]$ and $y[n]$ using the `subplot` function.

c. Now assume that the order of the systems is reversed, so that $x[n]$ is input to $H_2$, the output $v[n]$ of $H_2$ is input to $H_1$, and $y[n]$ is the output of $H_1$. Plot $v[n]$ and $y[n]$. Briefly explain why $v[n]$ is different in parts (b) and (c), whereas $y[n]$ is the same in both parts.