Homework 4 (Chapter 4)

Problems

1. Compute the Fourier transform of each of the following signals:
   
   a. \( x(t) = e^{-4|t|} \cos(5\pi t) \)
   
   b. \( x(t) = \begin{cases} 
   -\frac{1}{2}, & t \leq -\frac{1}{2} \\
   t, & -\frac{1}{2} < t \leq \frac{1}{2} \\
   \frac{1}{2}, & \frac{1}{2} < t 
   \end{cases} \)
   
   c. \( x(t) = \begin{cases} 
   2 + t^2, & 0 < t < 1 \\
   0, & \text{otherwise} 
   \end{cases} \)
   
   d. \( \left[ \frac{\sin \pi t}{\pi t} \right] \left[ \frac{\sin 2\pi (t-1)}{\pi (t-1)} \right] \)
   
   e. The signal \( x(t) \) depicted below:

   ![Signal Diagram]

2. Determine the continuous-time signal corresponding to each of the following transforms:
   
   a. \( X(j\omega) = e^{6j\omega} \frac{1}{(3 + j\omega)^2} \)
   
   b. \( X(j\omega) = \frac{1}{2 + 3j\omega - \omega^2} \)
   
   c. \( X(j\omega) = \frac{\sin^2(3\omega) \cos \omega}{\omega^2} \)

3. Assume that \( x(t) \) is purely imaginary and the Fourier transform of \( x(t) \) is \( X(j\omega) \)

   a. Prove \( X^*(j\omega) = -X(-j\omega) \)
   
   b. Determine whether the corresponding time-domain signal \( X(j\omega) \) is (i) real, imaginary or neither and (ii) even or odd or neither. Do this without evaluating the inverse Fourier transform of the given transform.
\[ X(j\omega) = \frac{\sin 2\omega}{\omega} e^{j(2\omega - \frac{\pi}{2})} \]

4. Determine which, if any, of the real signals depicted in below have Fourier transforms that satisfy each of the following conditions:

1. \( \text{Re}\{X(j\omega)\} = 0 \)
2. \( \text{Im}\{X(j\omega)\} = 0 \)
3. There exists a real \( \alpha \) such that \( e^{j\alpha\omega}X(j\omega) \) is real
4. \( \int_{-\infty}^{\infty} X(j\omega)d\omega = 0 \)
5. \( \int_{-\infty}^{\infty} \omega X(j\omega)d\omega = 0 \)
6. \( X(j\omega) \) is periodic

5. Suppose \( g(t) = x(t) \cos t \) and the Fourier transform of the \( g(t) \) is

\[ G(j\omega) = \begin{cases} 1, & |\omega| \leq 2 \\ 0, & \text{otherwise} \end{cases} \]

a. Determine \( x(t) \).

b. Specify the Fourier transform \( X_1(j\omega) \) of a signal \( x_1(t) \) such that
6. The input and the output of a causal LTI system are related by the differential equation

\[
\frac{d^2 y(t)}{dt^2} + 6 \frac{dy(t)}{dt} + 8y(t) = 2x(t)
\]

a. Find the impulse response of this system.
b. What is the response of this system if \( x(t) = te^{-2t}u(t) \)?
c. Repeat part (a) for the causal LTI system described by the equation

\[
\frac{d^2 y(t)}{dt^2} + \sqrt{2} \frac{dy(t)}{dt} + y(t) = 2 \frac{d^2 x(t)}{dt^2} - 2x(t)
\]

7. a. Determine the energy in the signal \( x(t) \) for which the Fourier transform \( X(j\omega) \) is depicted below.

\[
\begin{array}{c}
\text{X(}j\omega\text{)} \\
2 \\
1 \\
-1 \\
-2 \\
\hline
\omega
\end{array}
\]

b. Find the inverse Fourier transform of \( X(j\omega) \) of part (a).

8. Consider the impulse train

\[
p(t) = \sum_{k=-\infty}^{\infty} \delta(t - kT)
\]

a. Find the Fourier series of \( p(t) \).
b. Find the Fourier transform of \( p(t) \).
c. Consider the signal \( x(t) \) that depicted below where \( T_1 < T \).
show that the periodic signal $\bar{x}(t)$, formed by periodically repeating $x(t)$, satisfies.

$$\bar{x} = x(t) \ast p(t)$$