1. By knowing Kolmogorov axioms prove this property:
\[ P(A \cup B) = P(A) + P(B) - P(A \cap B). \]

2. A group of 10 people orders 5 teas, 3 coffees and 2 cakes in a restaurant (each of them orders one item and there is only one flavour of tea, cofee, and cake available). The absent-minded waiter forgets who ordered what, and hence, he randomly gives to each person an item. What is the probability that everybody gets what he/she had wanted.

3. A group of 6 men and 6 women is randomly divided into 2 groups of size 6 each. What is the probability that both groups will have the same number of men?

4. Pepys wrote Newton to ask which of three events is more likely: that a person get
   (a) at least 1 six when 6 dice are rolled,
   (b) at least 2 sixes when 12 dice are rolled, or
   (c) at least 3 sixes when 18 dice are rolled.
   What is the answer?

5. A pair of dice is rolled until a sum of either 5 or 7 appears. Find the probability that a 5 occurs first.
   Hint: Let \( E_n \) denote the event that a 5 occurs on the \( n \)th roll and no 5 or 7 occurs on the first \( n - 1 \) rolls. Compute \( P(E_n) \) and argue that \( \sum_{n=1}^{\infty} P(E_n) \) is the desired probability.

6. Suppose that you want to output 0 with probability \( \frac{1}{2} \) and 1 with probability \( \frac{1}{2} \). At your disposal is a procedure BIASED-RANDOM, that outputs either 0 or 1. It outputs 1 with some probability \( p \) and 0 with probability \( 1 - p \), where \( 0 < p < 1 \), but you do not know what \( p \) is. Give an algorithm that uses BIASED-RANDOM as a subroutine, and returns an unbiased answer, returning 0 with probability \( \frac{1}{2} \) and 1 with probability \( \frac{1}{2} \).

7. A closet contains 10 pairs of shoes. If 8 shoes are randomly selected, what is the probability that there will be
   (a) no complete pair?
   (b) exactly 1 complete pair?

8. If we know that \( P(A|C) \geq P(B|C), P(A|C^c) \geq P(B|C^c) \) prove that \( P(A) \geq P(B) \).

9. A total of \( n \) balls are sequentially and randomly chosen, without replacement, from an urn containing \( r \) red and \( b \) blue balls \( (n \leq r + b) \). Given that \( k \) of the \( n \) balls are blue, what is the conditional probability that the first ball chosen is blue.