1. You have three coins, one is symmetric, another has tails on both sides, and the last one will come up tails in \( \frac{3}{4} \) of times. you take a coin at random and flip it and it comes up tails. how probable is it that you had chosen the first coin (the symmetric one)?

2. Ali has lost his pen. he know that there’s a 0.75 probability that his pen is in point A and a 0.25 probability that it is in point B. but there’s a catch, even if he searches for his pen in point A (when the pen actually is in point A), he will find it with probability 0.3 but if it’s in point B and he searches there, he will find it with probability 0.9, and he doesn’t have the time to search in both places.
   (a) If he wants to maximize his chances of finding his pen, which point should he search in?
   (b) Suppose he tosses a coin and based on it’s outcome he chooses either point A or point B. if you know that he did actually find his pen, what is the probability that he did search in point B?

3. Suppose A, B and C are three events. for each case either prove the equation or provide and example in which it’s not true:
   (a) \( P(A \cup B|D) = P(A|D) + P(B|D) - P(A \cap B|D) \)
   (b) suppose A and B are independent: \( P(AB|C) = P(A|C)P(B|C) \)

4. There’s a bomb in the middle of computer department’s lobby. there are two wires on the bomb a red and a blue one and you must cut one of them to disarm the bomb. you somehow know that the probability that a person chooses the correct wire is \( p \). so to disarm the bomb you have two options. one, you choose one person and that person has to cut one of the wires. two, you create a poll and everyone ask to vote for the wire they want to cut and choose that one (there are an odd number of people present). which strategy is better? does it depend on the value of \( p \)?

5. You give a rubik’s cube to a monkey and it start to play with it. what is the probability that sometime in the future, the monkey actually solves it? (suppose the monkey randomly chooses to do something each time)

6. (a) Show that, if \( a \leq x(t) \leq b \) for every \( t \in S \), then \( F(x) = 1 \) for \( x > b \) and \( F(x) = 0 \) for \( x < a \).
   (b) Show that, if \( x(t) \leq y(t) \) for every \( t \in S \), then \( F_x(w) \geq F_y(w) \) for every \( w \).

7. (a) Suppose Y is a Binomial random variable with \( n \) trials. find \( k \) such that maximizes \( P(Y = k) \).
   (b) Suppose X is a Poisson random variable with parameter (mean) \( \lambda \). find \( t \) such that maximizes \( P(X = t) \).
   (remember that probability mass function of a Poisson random variable is given by the equation below)
   \[
   P(X = t) = \frac{\lambda^t e^{-\lambda}}{k!}
   \]
8. You are stuck in a mine field and you know that the number of mines in an area with a surface S is a Poisson random variable with mean $\lambda S$.

(a) What is the probability that the closest mine to you is just $d$ meters away?

(b) What is the probability that there are at least $n$ mines in a distance of $d$ meters from you? (closed form formula is not required)