Homework 4

Problems

1. we have the Joint Density function of two RV’s X and Y as shown below

\[ f(x, y) = Ce^{-(x^2+y^2)} \]

(a) Calculate the Constant C
(b) Calculate the Probability of the Region

\[ D = \{X \geq 0, Y \geq 0\} \]

2. Prove or disprove the following statements

(a) if two RV’s X and Y are independent then we have

\[ f_{x,y}(x, y) = f_x(x)f_y(y) \]

(b) if we have the following condition

\[ f_{x,y}(x, y) = f_x(x)f_y(y) \]

then the two RV’s X and Y are independent

(c) we have two independent RV’s X and Y and also we have

\[ Z = g(X) \quad W = h(Y) \]

so Z and W are also independent

3. Consider the Following Exponential Distribution

\[ f_x(x) = \begin{cases} \text{Ae}^{-Bx} & \text{if } x \geq 0 \\ 0 & \text{if } x \leq 0 \end{cases} \]

(a) Calculate A
(b) for which Constants(A,B) this distribution is Memoryless? a Distribution is Memoryless if

\[ P\{X > t + s|X > s\} = P\{X > t\} \]

you can consider that s,t are positive

4. Consider the Following Geometric Distribution

\[ P\{X = n\} = p(1-p)^{n-1} \quad n = 1, 2, 3, \ldots \]

show that this Distribution is Memoryless and also its converse(if X is a nonnegative integer valued RV satisfying condition of being Memoryless then then X is a Geometric RV)
5. Consider we are attending a Lottery and the probability of buying a winning ticket is 0.001

(a) if someone buy 1000 tickets what is the probability of buying at least one winning ticket?
(b) how many tickets are needed to be bought in order to be at least 95 percent confident about
buying at least one winning ticket?

**hint:** use Poisson Distribution

6. We have a Joint Distribution of two RV’s X and Y now prove Following Statements

(a)
\[
f_x(x) = \int_{-\infty}^{+\infty} f_{x,y}(x, \beta) d\beta
\]
\[
f_y(y) = \int_{-\infty}^{+\infty} f_{x,y}(\alpha, y) d\alpha
\]

(b)
\[
\frac{\partial F(x,y)}{\partial x} = \int_{-\infty}^{y} f_{x,y}(x, \beta) d\beta
\]
\[
\frac{\partial F(x,y)}{\partial y} = \int_{-\infty}^{x} f_{x,y}(\alpha, y) d\alpha
\]

(c)
\[
\lim_{y \to \infty} \frac{\partial F(x,y)}{\partial x} = f_x(x)
\]
\[
\lim_{x \to \infty} \frac{\partial F(x,y)}{\partial y} = f_y(y)
\]

**hint:** you may need Leibnitz integral rule