1. Prove for any collection of events $A_1, \ldots, A_n$:

$$P\left(\bigcup_{i=1}^{n} A_i\right) \leq \sum_{i=1}^{n} P(A_i)$$

2. Propose a method to simulate a fair coin using an unfair coin.

3. A group of 12 people orders 5 teas, 3 coffees and 4 cakes in a restaurant (each of them orders one item and there is only one flavour of tea, coffee, and cake available). The absent-minded waiter forgets who ordered what, and hence, he randomly gives to each person an item. What is the probability that everybody gets what he/she had wanted.

4. A group of 6 men and 6 women is randomly divided into 2 groups of size 6 each. What is the probability that both groups will have the same number of men?

5. A pair of dice is rolled until a sum of either 5 or 7 appears. Find the probability that a 5 occurs first.

   Hint: Let $E_n$ denote the event that a 5 occurs on the $n$th roll and no 5 or 7 occurs on the first $n-1$ rolls. Compute $P(E_n)$ and argue that $\sum_{n=1}^{\infty} P(E_n)$ is the desired probability.

6. If we know that $P(A|C) \geq P(B|C)$, $P(A|C^c) \geq P(B|C^c)$ prove that $P(A) \geq P(B)$.

7. If $n$ people are seated in a random manner in a row containing $2n$ seats, what is the probability that no two people will occupy adjacent seats?

8. A bowl contains 10 balls numbered 1 through 10. Four balls are drawn without replacement. What is the probability that the second largest number drawn will be 6?

9. Suppose that 10 cards, of which five are red and five are green, are put at random into 10 envelopes, of which seven are red and three are green, so that each envelope contains one card. Determine the probability that exactly $k$ envelopes will contain a card with a matching color ($k = 0, 1, \ldots, 10$).