1. Consider the signals \( x(t) \) and \( y(t) \) shown below:

![Signal Diagram](image)

Sketch carefully each of the following signals: (Just answer 4 of 5)

(a) \( y(t - 1)x(t - 1) \)
(b) \( y(3 - \frac{t}{2}) \)
(c) \( x \left( \frac{3t}{2} \right) \delta(t + 1) \)
(d) \( 2x(t)u(t) + 3x(-t)u(-t) \)
(e) \( y(t) \left[ \delta \left( t + \frac{1}{2} \right) - \delta \left( t - \frac{3}{2} \right) \right] \)

2. Show the following:

(a) \( g(t)\delta(t - a) = g(a)\delta(t - a) \)
(b) \( g(t)\delta'(t) = g(0)\delta'(t) - g'(0)\delta(t) \)

Then evaluate the following integrals: (Just Answer 2 of 3)

(a) \( \int_{-\infty}^{+\infty} (t^2 + 4t - 5)\delta'(t + 1) \, dt \)
(b) \( \int_{-\infty}^{+\infty} 2t^3\delta'(t - 6) \, dt \)
(c) \( \int_{-2}^{+2} 3t^2[\delta(t) + 4\delta(t + 1) + \delta(t - 3)] \, dt \)

3. Show that \( \int_{-\infty}^{+\infty} x^2(t) \, dt = \int_{-\infty}^{+\infty} x_e^2(t) \, dt + \int_{-\infty}^{+\infty} x_o^2(t) \, dt \) Where \( x_e(t) \) and \( x_o(t) \) are the even and odd parts of a signal \( x(t) \).

4. The following systems are described by their input-output relationship. Determine necessary and sufficient conditions on the functions or parameters involved in the system definition so the system is 1) memoryless, 2) causal, 3) invertible, 4) stable, 5) time-invariant, 6) linear. (Just answer 3 of 4)

(a) Suppose \( f: \mathbb{R} \rightarrow \mathbb{R} \) is a function, The system is defined by

\[
y(t) = f(x(t)).
\]
(b) Suppose \( g: \mathbb{R} \to \mathbb{R} \) is a function. The system is defined by
\[
y(t) = x(g(t)).
\]

(c) Suppose \( a \) and \( b \) are two real numbers. The system is defined by the following differential equation.
\[
\begin{cases}
y'(t) + ay(t) = x(t) \\
y(0) = b
\end{cases}
\]
For simplicity, you may assume that every input to this system is continuous.

(d) Suppose \( h: \mathbb{R} \to \mathbb{R} \) is a continuous function. The system is defined by
\[
y(t) = \int_0^t h(u) \, du
\]
Note: you have to investigate each of the mentioned properties for each system independently. For instance, for the first system, you have to determine under what conditions on \( f \), the system will be linear, and so on.

5. Determine whether or not of the following signals is periodic. If a signal is periodic, determine its fundamental period. (Just answer 2 of first 4 and (e))
(a) \( x_1(t) = 1 + \sin^2(20t) \cos \left(30t + \frac{n}{3}\right) \)
(b) \( x_2(t) = \sin(1 + 8t^2) \cos(4t^2) \)
(c) \( x_3(t) = e^{\frac{-t}{5}} + e^\frac{-t}{5} \)
(d) \( x_4(t) = \sin(|t|) \cos(|t|) \)
(e) \( x_5[n] = \cos(\frac{n\pi}{4}) \cos(n\pi/4) \)

6. Determine which of the properties (memoryless, time-invariant, linear, causal, stable) hold and which do not hold for each of the following systems. Justify your answers. (Just answer 5 of 8)
(a) \( y(t) = \cos(10\pi t) x(t) \)
(b) \( y(t) = x(t - 1) + x(1 - t) \)
(c) \( y(t) = \begin{cases} 
0, & t < 0 \\
-x(t) + x(t-1), & t \geq 0
\end{cases} \)
(d) \( y(t) = \begin{cases} 
0, & x(t) < 0 \\
x(t-4) + x(t), & x(t) \geq 0
\end{cases} \)
(e) \( y(t) = x(t) u(t) \)
(f) \( y(t) = x(t) + 2x'(t) \)
(g) \( y(t) = \cos(100\pi t) + \int_{-\infty}^{t} x(\tau) \, d\tau \)
(h) \( y(t) = \frac{d}{dt} \{ e^{-t} x(t) \} \)

7. The convolutional result of \( x(t) \) and \( y(t) \) are given in the form below:
\[
x(t) \ast y(t) = \int (1 - |u|) x(t - u) \, du
\]
What is \( y(t) \)?

8. Determine if each of the following systems is invertible. If it is, construct the inverse system. If it is not find two input signals to the system that have the same output. (Just answer 3 of 4)
(a) \( y(t) = x(t - 3) \)
(b) \( y(t) = \frac{dx(t)}{dt} \)

(c) \( y(t) = \int_{-\infty}^{t} x(u) du \)

(d) \( y(t) = \int e^{-2(t-u)} x(u) du \)

9. For the following LTI systems with impulse response \( h(t) \) (or \( h[n] \)), compute the output for the given input, and determine if the system is stable, causal, invertible. (Just answer 3 of 4)

(a) \( h(t) = \delta(t - T) + \delta(t) + \delta(t + T) \) and \( x(t) = \begin{cases} 1, & |t| \leq 10 \\ 0, & |t| > 10 \end{cases} \) (Solve for \( T = 1, 2, 3 \))

(b) \( h(t) = u(t + a) - u(t - a) \), where \( a > 0 \) and the input is the same as in (a).

(c) \( h(t) = e^{-at} u(t) \), where \( a > 0 \) and \( x(t) = u(t + 12) - u(t - 12) \).

(d) \( h[n] = \alpha^n u[n] \) and \( x[n] = \beta^n u[n] \).

10. Determine whether each of the following statements concerning LTI systems is true or false. Justify your answers. (Just answer 3 of 4)

(a) If an LTI system is causal, it is stable.

(b) The inverse of a causal LTI system is always causal.

(c) The cascade of a non-causal LTI system with a causal one is necessarily non causal.

(d) If \( h(t) \) is the impulse response of an LTI system and it is periodic and nonzero, the system is unstable.