1. Let

\[ x[n] = (-1)^n u[n] + \alpha^n u[-n - n_0]. \]

Determine the constraints on the complex number \( \alpha \) and the integer \( n_0 \), given that the ROC of \( X(z) \) is

\[ 1 < |z| < 2. \]

2. Consider the signal

\[ x[n] = \left( \frac{1}{5} \right)^n u[n - 3]. \]

Use eq. (10.3) to evaluate the z-transform of this signal, and specify the corresponding region of convergence.

3. Determine the unilateral z-transform of each of the following signals, and specify the corresponding regions of convergence:

(a) \( x_1[n] = \left( \frac{1}{4} \right)^n u[n + 5] \)

(b) \( x_2[n] = \delta[n + 3] + \delta[n] + 2^n u[-n] \)

4. Following are several z-transforms. For each one, determine the inverse z-transform using both the method based on the partial-fraction expansion and the Taylor’s series method based on the use of long division.

\[ X(z) = \frac{1 - z^{-1}}{1 - \frac{1}{4} z^{-2}}, \quad |z| > \frac{1}{2}. \]

5.
A right-sided sequence $x[n]$ has $z$-transform

$$X(z) = \frac{3z^{-10} + z^{-7} - 5z^{-2} + 4z^{-1} + 1}{z^{-10} - 5z^{-7} + z^{-3}}.$$ 

Determine $x[n]$ for $n < 0$.

6.
Consider the signal

$$x[n] = \begin{cases} 
(\frac{1}{3})^n \cos(\frac{\pi}{4} n), & n \leq 0 \\
0, & n > 0 
\end{cases}$$

Determine the poles and ROC for $X(z)$. 