Weekly Homework 8
Dr. Sameti
Signals and Systems
Communication Systems

Q1:

Let $x[n]$ be a discrete-time signal with spectrum $X(e^{j\omega})$, and let $p(t)$ be a continuous-time pulse function with spectrum $P(j\omega)$. We form the signal

$$y(t) = \sum_{n=-\infty}^{+\infty} x[n]p(t - n).$$

a) Determine the spectrum $Y(j\omega)$ in terms of $X(e^{j\omega})$ and $P(j\omega)$.

b) If

$$p(t) = \begin{cases} 
\cos 8\pi t, & 0 \leq t \leq 1 \\
0, & \text{elsewhere}
\end{cases}$$

Determine $P(j\omega)$ and $Y(j\omega)$. 
Q2:

A class of popularly used pulses in PAM are those which have a raised frequency response. The frequency response of one of the members of this class is

\[ P(j\omega) = \begin{cases} \frac{1}{2} \left(1 + \cos \frac{\omega T_1}{2}\right), & 0 \leq |\omega| \leq \frac{2\pi}{T_1}, \\ 0, & \text{elsewhere} \end{cases} \]

Where \( T_1 \) is the intersymbol spacing.
Determine \( p(0) \).

Q3:

Let \( x(t) \) be a real-valued signal for which \( X(j\omega) = 0 \) when \( |\omega| > 2000\pi \).
Amplitude modulation is performed to produce the signal

\[ g(t) = x(t) \sin(2000\pi t). \]

A proposed demodulation technique is illustrate in Figure 1 where \( g(t) \) is the input, \( y(t) \) is the output, and the ideal lowpass filter has cutoff frequency \( 2000\pi \) and passband gain of 2. Determine \( y(t) \).

![Figure 1](image)
Q4:

Suppose

\[ x(t) = \sin 200\pi t + 2 \sin 400\pi t \]

And

\[ g(t) = x(t) \sin 400\pi t. \]

If the product \( g(t) \sin 400nt \) is passed through an ideal lowpass filter with cutoff frequency \( 400\pi \) and passband gain of 2, determine the signal obtained at the output of the lowpass filter.

Q5:

For what values of \( \omega_0 \) in the range \(-\pi < \omega_0 \leq \pi \) is amplitude modulation with carrier \( e^{j\omega_0 t} \) equivalent to amplitude modulation with carrier \( \cos \omega_0 t \)?

Q6:

Suppose \( x[n] \) is a real-valued discrete-time signal whose Fourier transform \( X(e^{j\omega}) \) has the property that

\[ X(e^{j\omega}) = 0 \quad \text{for} \quad \frac{\pi}{8} \leq \omega \leq \pi. \]

We use \( x[n] \) to modulate a sinusoidal carrier \( c[n] = \sin(5\pi/2)n \) to produce

\[ y[n] = x[n]c[n]. \]

Determine the values of \( \omega \) in the range \( 0 \leq \omega \leq \pi \) for which \( Y(e^{j\omega}) \) is guaranteed to be zero.