Weekly Homework

Dr. Sameti
CE 40242: Signals and Systems

Problem 1.

Consider a continuous-time ideal bandpass filter whose frequency response is

\[ H(j\Omega) = \begin{cases} 1 & \omega \leq |\omega| \leq 3\omega_c \\ 0 & \text{elsewhere} \end{cases} \]

a: If \( h(t) \) is the impulse response of this filter, determine a function \( g(t) \) such that

\[ h(t) = (\frac{\sin \omega_c t}{\pi t})g(t) \]

b: As \( \omega_c \) is increased, does the impulse response of the filter get more concentrated or less concentrated about the origin?

Problem 2.

Consider a continuous-time causal and stable LTI system whose input \( x(t) \) and output \( y(t) \) are related by the differential equation

\[ \frac{dy(t)}{dt} + 5y(t) = 2x(t) \]

what is the final value \( s(\infty) \) of the step response \( s(t) \) of this filter? Also, determine the value of the \( t_0 \) for which

\[ s(t_0) = s(\infty)[1 - \frac{1}{e^2}] \]
Problem 3.

A particular first-order causal and stable discrete-time LTI system has a step response whose maximum overshoot is 50% of its final value. If the final value is 1, determine a difference equation relating the input $x[n]$ and output $y[n]$ of this filter.

Problem 4.

By computing the group delay at two selected frequencies, verify that each of the following frequency response has nonlinear phase.

- **a:** $H(j\omega) = \frac{1}{j\omega + 1}$
- **b:** $H(j\omega) = \frac{1}{(j\omega + 1)^2}$
- **c:** $H(j\omega) = \frac{1}{(j\omega + 1)(j\omega + 2)}$

Problem 5.

Consider a discrete-time lowpass filter whose impulse response $h[n]$ is known to be real and whose frequency response magnitude in the region $-\pi \leq \omega \leq \pi$ is given as:

$$|H(e^{j\omega})| = \begin{cases} 1 & |\omega| \leq \frac{\pi}{4} \\ 0 & \text{otherwise} \end{cases}$$

Determine and sketch the real-value impulse response $h[n]$ for this filter when the corresponding group delay function is specified as:

- **a:** $\tau(\omega) = 5$
- **b:** $\tau(\omega) = \frac{5}{2}$
- **c:** $\tau(\omega) = -\frac{5}{2}$

Problem 6.

Consider a four-point, moving-average, discrete-time filter for which the difference equation is

$$y[n] = b_0 x[n] + b_1 x[n-1] + b_2 x[n-2] + b_3 x[n-2]$$

Determine and sketch the magnitude of the frequency response for each of the following cases:

- **a:** $b_0 = b_3 = 0, b_1 = b_2$
- **b:** $b_1 = b_2 = 0, b_0 = b_3$

Problem 7.

A particular causal LTI system is described by the difference equation

$$y[n] - \frac{\sqrt{2}}{2} y[n-1] + \frac{1}{4} y[n-2] = x[n] - x[n-1]$$

Find the impulse response of this system.
Problem 8.

Problem 6.30 from your textbook second edition.