1- Determine the function of time, \( x(t) \), for each of the following Laplace transforms and their associated region of convergence:

a) \( \frac{1}{s^2+9} \), \( \text{Re}\{s\} > 0 \)

b) \( \frac{s}{s^2+9} \), \( \text{Re}\{s\} < 0 \)

c) \( \frac{s+1}{(s+1)^2+9} \), \( \text{Re}\{s\} < -1 \)

d) \( \frac{s+2}{s^2+7s+12} \), \(-4 < \text{Re}\{s\} < -3 \)

e) \( \frac{s+1}{s^2+5s+6} \), \(-3 < \text{Re}\{s\} < -2 \)

f) \( \frac{(s+1)^2}{s^2-s+1} \), \( \text{Re}\{s\} > \frac{1}{2} \)

g) \( \frac{s^2-s+1}{(s+1)^2} \), \( \text{Re}\{s\} > -1 \)

2- Consider an LTI system for which the system function \( H(s) \) has the pole-zero pattern shown below.

![Pole-zero diagram]

a) Indicate all possible ROCs that can be associated with this pole-zero pattern.

b) For each ROC identified in part (a), specify whether the associated system is stable and/or causal.

3- The system function of a causal LTI system is

\[
H(s) = \frac{s + 1}{s^2 + 2s + 2}
\]

Determine the response \( y(t) \) when the input is

\[
x(t) = e^{-|t|}, \quad -\infty < t < \infty
\]
4- We are given the following five facts about a real signal \( x(t) \) with Laplace transform \( X(s) \).

1. \( X(s) \) has exactly two poles.
2. \( X(s) \) has no zeros in the finite \( s \)-plane.
3. \( X(s) \) has a pole at \( s = -1 + j \).
4. \( e^{2t}x(t) \) is not absolutely integrable.
5. \( X(0) = 8 \).

Determine \( X(s) \) and specify its region of convergence.

5- Consider a continuous time LTI system for which the input \( x(t) \) and output \( y(t) \) are related by the difference equation

\[
\frac{d^2 y(t)}{dt^2} - \frac{dy(t)}{dt} - 2y(t) = x(t)
\]

Let \( X(s) \) and \( Y(s) \) denote the Laplace transforms of \( x(t) \) and \( y(t) \) respectively and let \( H(s) \) denote the Laplace transform of \( h(t) \), the system impulse response.

a) Determine \( H(s) \) as a ratio of two polynomials in \( s \). Sketch the pole-zero pattern of \( H(s) \).

b) Determine \( h(t) \) for each of the following cases:
   i. The system is stable.
   ii. The system is causal.
   iii. The system is neither stable nor causal.

6-

a) Show that if \( x(t) \) is an even function so that \( x(t) = x(-t) \) then \( X(s) = X(-s) \).

b) Show that if \( x(t) \) is an odd function so that \( x(t) = -x(-t) \) then \( X(s) = -X(-s) \).

c) Determine which if any of the pole-zero plots in figure could correspond to an even function of time. For those that could, indicate the required ROC.
The signal

\[ y(t) = e^{-2t}u(t) \]

is the output of a causal all-pass system for which the system function is

\[ H(s) = \frac{s - 1}{s + 1} \]

a) Find and sketch at least two possible inputs \( x(t) \) that could produce \( y(t) \).

b) What is the input \( x(t) \) if it is known that

\[ \int_{-\infty}^{\infty} |x(t)| dt < \infty \]

Consider the system S characterized by the differential equation

\[ \frac{d^3 y(t)}{dt^3} + 6 \frac{d^2 y(t)}{dt^2} + 11 \frac{dy(t)}{dt} + 6y(t) = x(t) \]
a) Determine the zero-state response of this system for the input $x(t) = e^{-4t}u(t)$.

b) Determine the zero-input response of the system for $t > 0^-$, given that

$$y(0^-) = 1, \quad \frac{dy(t)}{dt} |_{t=0} = -1, \quad \frac{d^2y(t)}{dt^2} |_{t=0} = 1$$

c) Determine the output of S when the input is $x(t) = e^{-4t}u(t)$ and the initial conditions are the same as those specified in part (b).