Homework 1 (Estimation theory)

1. Let $X_1, \ldots, X_n$ be a random sample from a gamma $(\alpha, \beta)$ population. Find a two dimensional sufficient statistic for $(\alpha, \beta)$.

2. Suppose $X_1, X_2, \ldots, X_n$ be iid with double exponential distribution. find a minimal sufficient statistic for $\theta$.

$$f(x | \theta) = \frac{1}{2} e^{-|x-\theta|}, \quad -\infty < x < \infty, \quad -\infty < \theta < \infty$$

3. For each of the following pdfs let $X_1, \ldots, X_n$ be iid observations. Find a complete sufficient statistic, or show that one does not exist.

   (a) $f(x | \theta) = \frac{2x}{\theta^2}, \quad 0 < x < \theta, \quad \theta > 0$

   (b) $f(x | \theta) = \frac{\theta}{(1+x)^{\frac{\theta}{\theta}+\theta}}, \quad 0 < x < \infty, \quad \theta > 0$

4. Suppose $X_1, X_2, \ldots, X_n$ be iid with Uniform $(0, \theta)$. $\max X_i$ is a sufficient statistic for $\theta$. Show that it is a complete sufficient statistic for $\theta$.

5. Let $X_1, \ldots, X_n$ be a random sample from a uniform distribution on the interval $(\theta, 1 + \theta)$ $\theta > 0$. Find a minimal sufficient statistic for $\theta$. Is the statistic complete?

6. Let $X_1, \ldots, X_n$ be a random sample from a population with pmf:

$$P_\theta(X = x) = \theta x (1 - \theta)^{1-x}, \quad x = 0 \text{ or } 1, \quad 0 \leq \theta \leq \frac{1}{2}$$

Find the method of moments estimator of $\theta^2$.

7. Let $X_1, \ldots, X_n$ be a random sample from the pdf:

$$f(x | \theta) = \theta x^{-2}, \quad 0 < \theta \leq x < \infty$$

find the MLE of $\theta$. 

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8. Let $X_1, \ldots, X_n$ be a sample from a population with double exponential:

$$f(x \mid \theta) = \frac{1}{2} e^{-|x-\theta|}, \quad -\infty < x < \infty, \quad -\infty < \theta < \infty$$

find the MLE of $\theta$.

9. Let $X_1, \ldots, X_n$ be a random sample from a Bernoulli ($\lambda$). let $\lambda$ have a $\text{Beta}(\alpha, \beta)$ distribution. assume we are using mean squared loss. find the Bayes estimator of $\lambda$. 

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