

# Polygonal Line Simplification

## Introduction

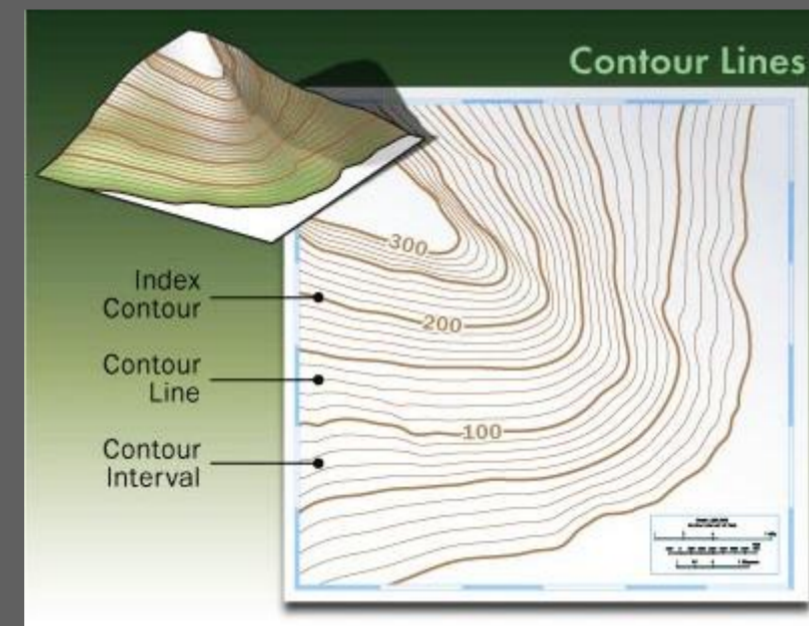
### Problem

Given a polygonal chain, find the optimal simplification of the polygonal chain efficiently.

### Applications

- Computational Geometry
- Geographic Information Systems (GIS)
- Digital Image Processing

E.g. border of countries, rivers and contours.



### Motivation

The computation and presentation of this data is very time consuming. Using simplification, we can reduce the total amount of input data and consequently reduce the computation time.

### Optimization Goals

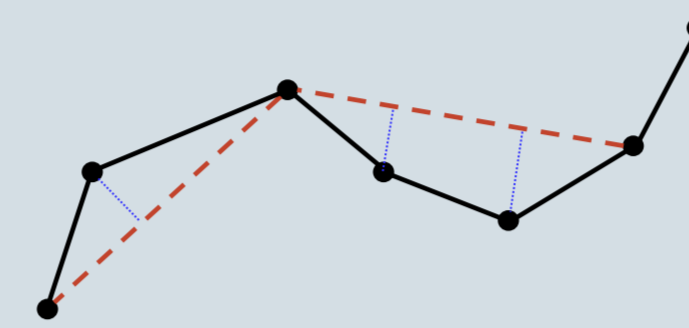
- For a given error, our goal is to find a path with the minimum number of vertices.
- For a given number  $k$ , the goal is to find a simplification of at most  $k$  vertices and with the minimum simplification error.



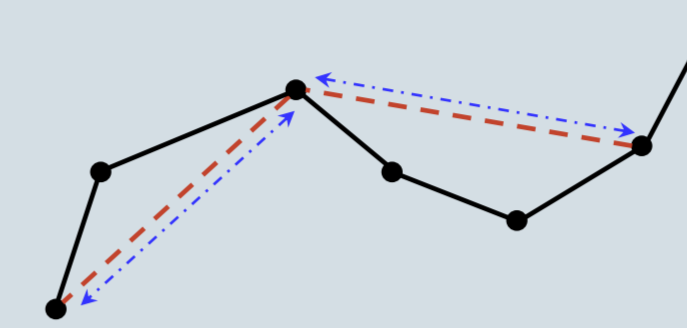
### Versions

- Restricted: the vertices of the simplified path should be a subsequence of the vertices of the original path.
- Unrestricted: There is no such restriction.

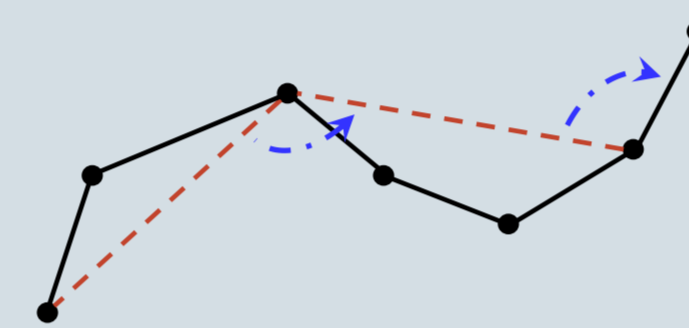
## Error Measures



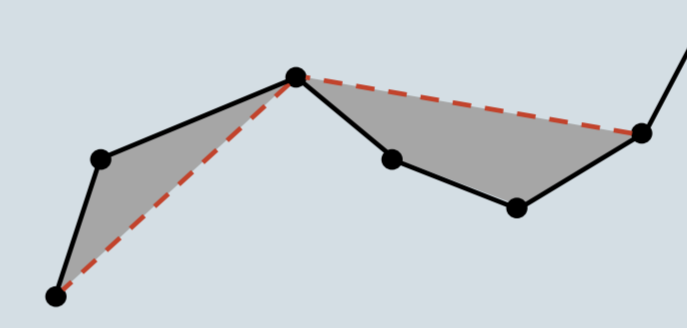
The Hausdorff Distance



The Retained Length



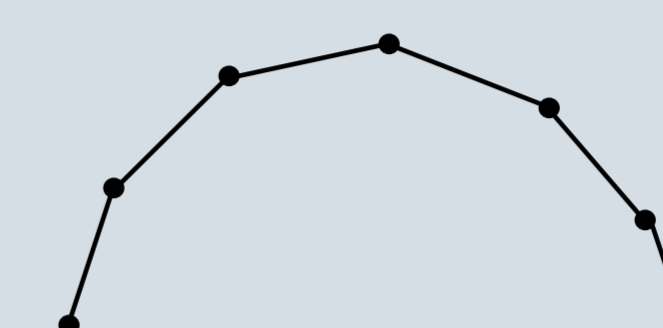
The Angle



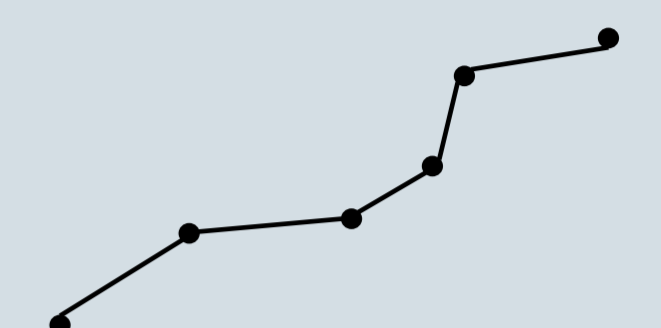
The Area Measure

Other measures: Fréchet, Uniform,  $L_1$ , ...

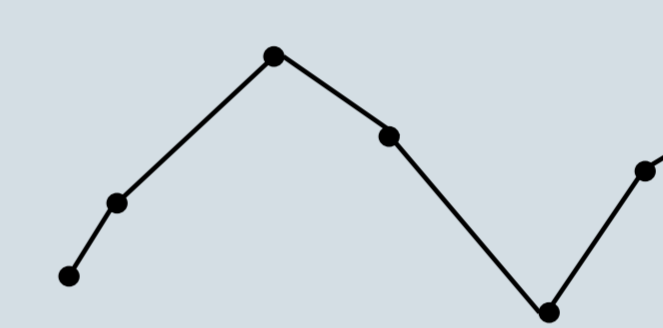
## Chain Types



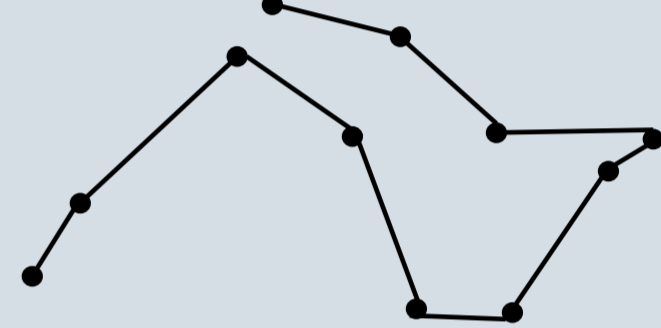
A Convex Path



An xy-monotone Path



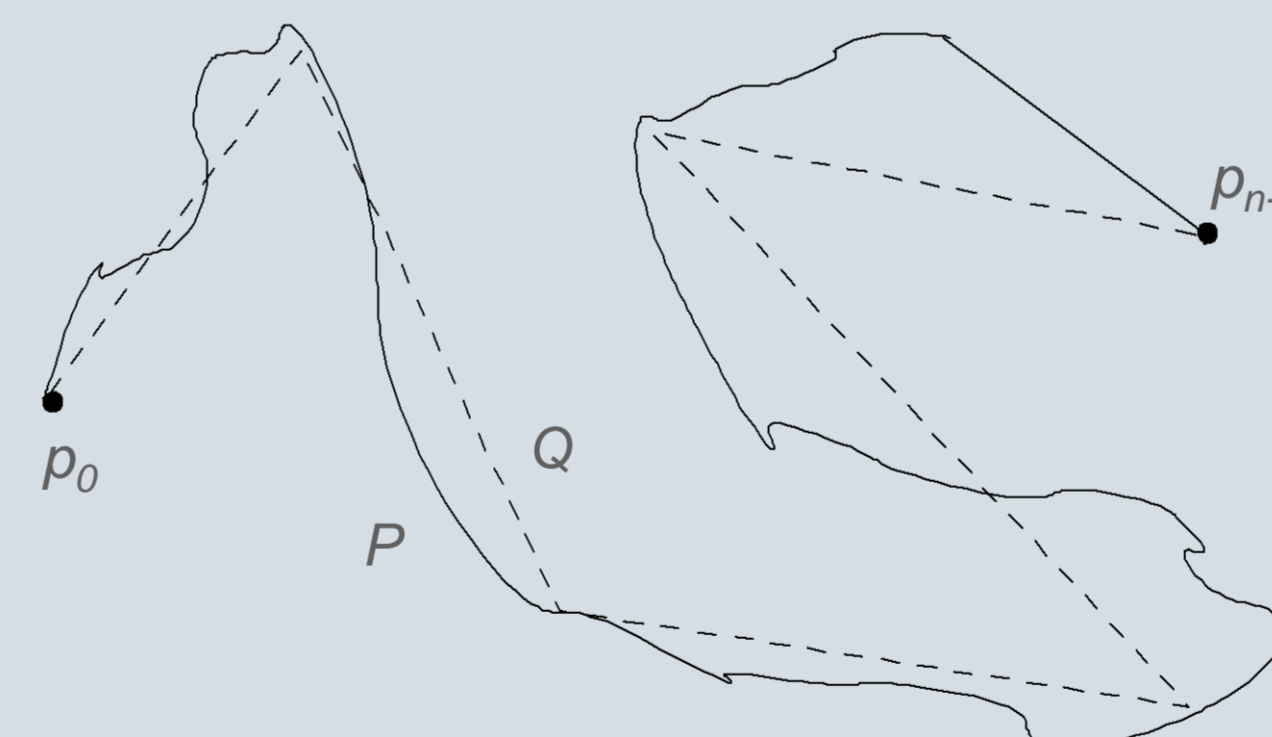
An x-monotone Path



General Path

## Minimum-Link Simplification

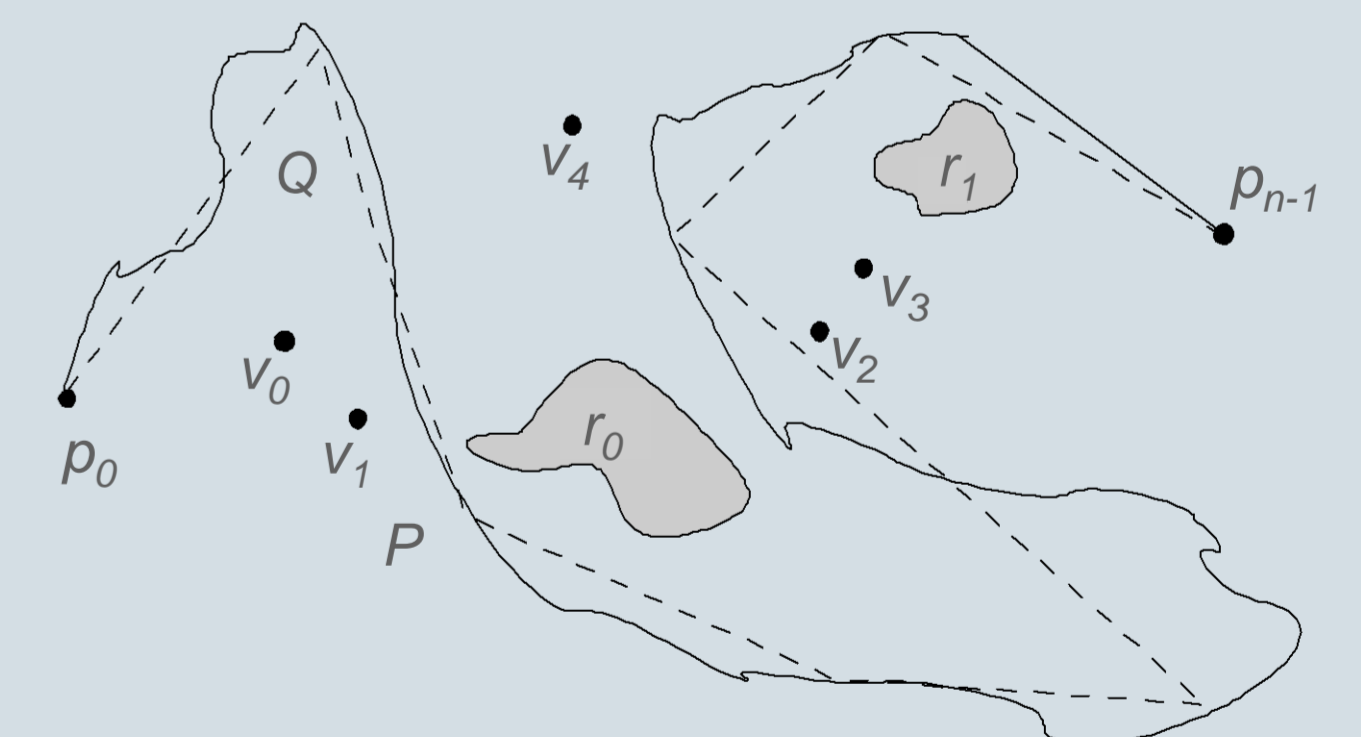
The min-link simplification problem has been extensively studied under different error measures for various input chain types.



An input chain  $P$  and a simplified path  $Q$ .

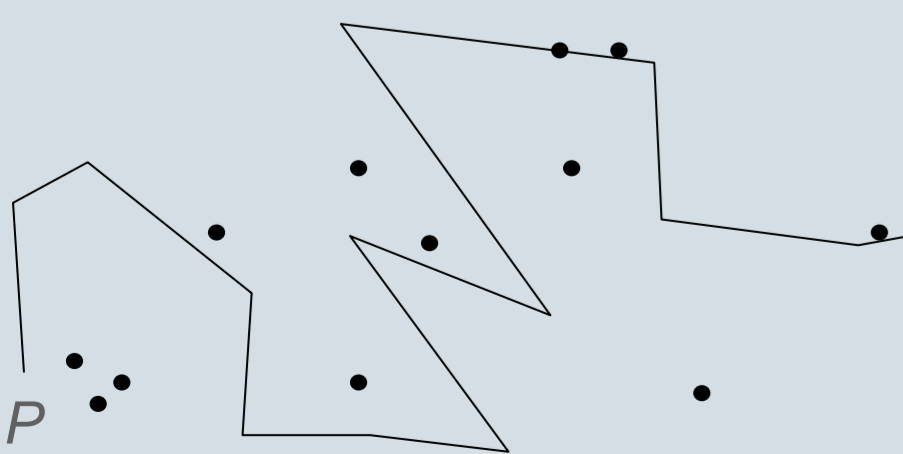
## Homotopic Simplification

There are few algorithms that preserve the homotopy in simplification.

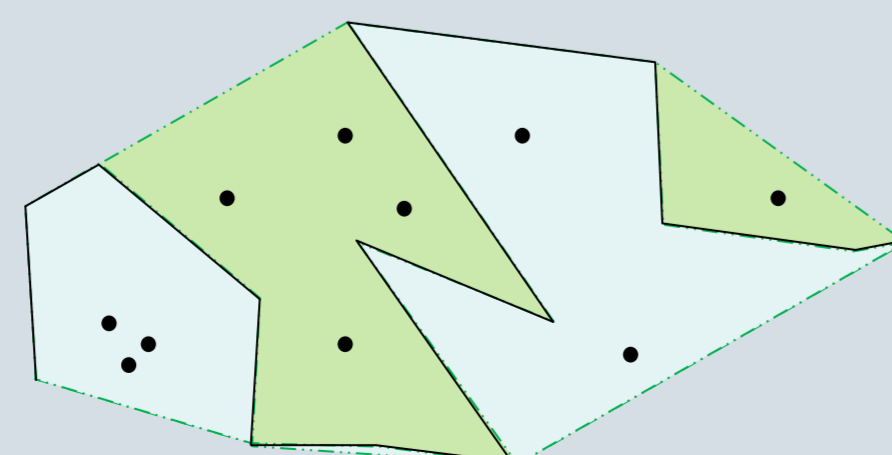


A homotopic min-link simplification  $Q$  in presence of obstacles.

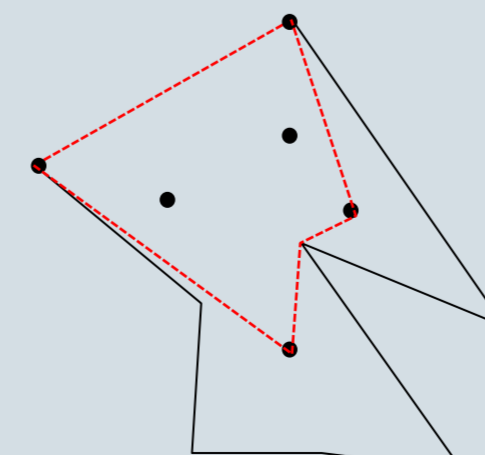
A given polygonal chain and a set of points in the plane.



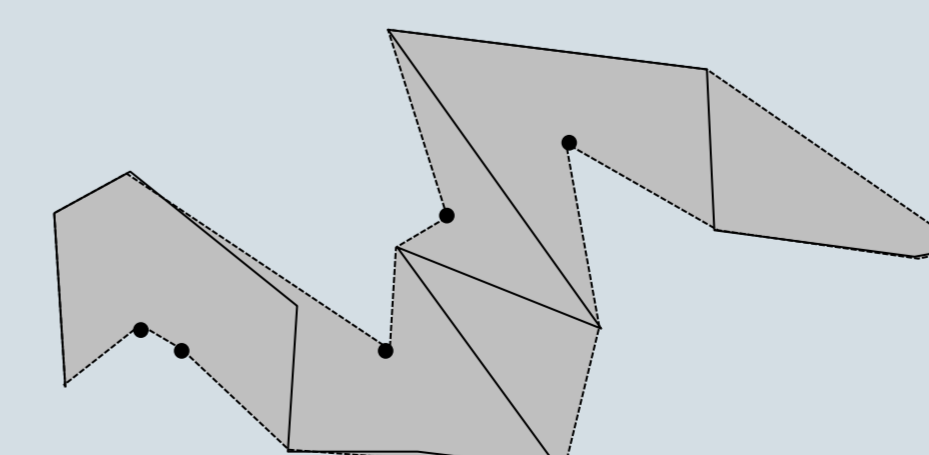
The area inside the convex hull of the input chain is divided into two parts.



For each polygon shaped between the input path and the convex hull, the *relative convex hull* is computed and removed.



A permitted region that all homotopic shortcuts lie inside is identified.



A graph containing all homotopic shortcuts with error less than a given error is built. The shortest path in this graph is the answer to the problem. [Daneshpajouh & Ghodsi'10].

