Homework 4 (Chapter 4)

Problems

1. Compute the Fourier transform of each of the following signals: (P 4.21 (a, c, f, g, i) p. 338 and 2 extra parts.)
   a. $x(t) = e^{-|t|} \cos 2t$
   b. $[e^{-\alpha t} \cos \omega_0 t]u(t), \alpha > 0$
   c. $x(t) = \begin{cases} 1 + \cos \pi t, & |t| \leq 1 \\ 0, & |t| > 1 \end{cases}$
   d. $\left[\frac{\sin \pi t}{\pi t}\right]\left[\frac{\sin 2\pi(t-1)}{\pi(t-1)}\right]$
   e. $x(t)$ as shown in Figure P4.21(a) in p. 338 of textbook.
   f. $x(t) = \begin{cases} 1 - t^2, & 0 < t < 1 \\ 0, & \text{otherwise} \end{cases}$
   g. The signal $x(t)$ depicted below:

2. Determine the continuous-time signal corresponding to each of the following transforms:
   a. $X(j\omega) = j[\delta(\omega + 1) - \delta(\omega - 1)] - 3[\delta(\omega - \pi) + \delta(\omega + \pi)]$
   b. $X(j\omega) = 2\sin(2\omega - \pi/2)$

3. Determine which, if any, of the real signals depicted in below have Fourier transforms that satisfy each of the following conditions:
   1. $\Re\{X(j\omega)\} = 0$
   2. $\Im\{X(j\omega)\} = 0$
   3. There exists a real $\alpha$ such that $e^{j\alpha \omega}X(j\omega)$ is real
   4. $\int_{-\infty}^{\infty} X(j\omega)d\omega = 0$
   5. $\int_{-\infty}^{\infty} \omega X(j\omega)d\omega = 0$
   6. $X(j\omega)$ is periodic
4. Let $X(j\omega)$ denote the Fourier transform of the signal $x(t)$ depicted in Figure P4.25 of textbook p. 341. (P 4.25 p. 341 with little change)

a. $X(j\omega)$ can be written as $A(j\omega)e^{j\theta(j\omega)}$ where $A(j\omega)$ and $\theta(\omega)$ are real. Find $\theta(j\omega)$.

b. Find $X(j0)$.

c. Find $\int_{-\infty}^{\infty} X(j\omega)d\omega$.

d. Evaluate $\int_{-\infty}^{\infty} X(j\omega)\frac{2\sin\omega}{\omega}e^{j2\omega}d\omega$.

e. Evaluate $\int_{-\infty}^{\infty} |X(j\omega)|^2d\omega$.

f. Sketch the inverse Fourier transform of $\text{Re}\{X(j\omega)\}$.

**Note:** You should perform all these calculations without explicitly evaluating $X(j\omega)$.

5. Find the impulse response of a system with the following frequency response: (P 4.18 p. 337)

$$H(j\omega) = \frac{(\sin^2(3\omega))\cos\omega}{\omega^2}.$$

6. A causal and stable LTI system $S$ has the following frequency response: (P 4.34 pp. 345 and 346)

$$H(j\omega) = \frac{j\omega+1}{6-\omega^2+3j\omega}.$$
a. Determine a differential equation relating the input $x(t)$ and output $y(t)$ of $S$.

b. Determine the impulse response $h(t)$ of $S$.

c. What is the output of $S$ when the input is $x(t) = e^{-4t}u(t) - te^{4t}u(t)$?

7. Consider the signal $x(t)$ in Figure P4.37 in p. 347 of textbook.

a. Find the Fourier transform $X(j\omega)$ of $x(t)$.

b. Sketch the signal $\bar{x}(t) = x(t) * \sum_{k=-\infty}^{\infty} \delta(t - 4k)$.

c. Find another signal $g(t)$ such that $g(t)$ is not the same as $x(t)$ and
$$\bar{x}(t) = g(t) * \sum_{k=-\infty}^{\infty} \delta(t - 4k).$$

d. Argue that, although $G(j\omega)$ is different from $X(j\omega)$, $G(j\pi/2k) = X(j\pi/2k)$ for all integers $k$. You should not explicitly evaluate $G(j\omega)$ to answer this question. (P4.37 pp. 346 and 347)

8. Let $g_1(t) = \{[\cos(\omega_0 t)]x(t)\} * h(t)$ and $g_2(t) = \{[\sin(\omega_0 t)]x(t)\} * h(t)$ where
$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk100t}$$
is a real-valued periodic signal that $h(t)$ is the impulse response of a stable LTI system.

a. Specify a value for $\omega_0$ and any necessary constraints on $H(j\omega)$ to ensure that $g_1(t) = \Re\{a_5\}$ and $g_2(t) = \Im\{a_5\}$.

b. Give an example of $h(t)$ such that $H(j\omega)$ satisfies the constraints you specified in part (a). (P4.42 p. 348)

**Practical Assignment**

1. Consider a discrete-time system $H_1$ with impulse response
$$h_1[n] = \delta[n] + \delta[n - 1] - \delta[n - 2] - \delta[n - 3],$$
a discrete-time system $H_2$ with impulse response
$$h_2[n] = \left(\frac{1}{2}\right)^n (u[n + 3] - u[n - 3]),$$
and a discrete-time signal
$$x[n] = \left(\frac{1}{4}\right)^n (u[n] - u[n - 6]).$$
The signals $h_1[n]$, $h_2[n]$, and $x[n]$ are all defined for $-8 \leq n \leq 8$.

a. Plot $h_1[n]$, $h_2[n]$, and $x[n]$ together using the `subplot` function.
b. Consider a system $H$ formed from the series connection of $H_1$ and $H_2$, where $x[n]$ is input to $H_1$, the output $v[n]$ of $H_1$ is input to $H_2$, and the output of $H_2$ is $y[n]$. Use the $\text{conv}$ function to find $v[n]$ and $y[n]$. Plot $v[n]$ and $y[n]$ using the $\text{subplot}$ function.

c. Now assume that the order of the systems is reversed, so that $x[n]$ is input to $H_2$, the output $v[n]$ of $H_2$ is input to $H_1$, and $y[n]$ is the output of $H_1$. Plot $v[n]$ and $y[n]$. Briefly explain why $v[n]$ is different in parts (b) and (c), whereas $y[n]$ is the same in both parts.