1. Let $X$ take on the values $-1, 0, 1, \ldots$ with probabilities

$$P(X = -1) = p, \ P(X = k) = (1 - p)^2 p^k, \ k = 0, 1, \ldots,$$

where $0 < p < 1$. Find the UMVUE for $p$.

2. If $T$ is a complete sufficient statistic with binomial distribution $b(p, n)$. Find a UMVU estimator for $pq$ (where $p$ is the prob. of success, and $q=1-p$)

Hint: if $E_{\theta} \delta(T) = g(\theta)$, then $\delta(T)$ is a UMVUE for $g(\theta)$

3. Suppose that $x_1, \ldots, x_n$ are iid according to the uniform distribution $U(0, \theta)$ and that $g(\theta) = \theta/2$

(a) Show that $T = x_{(n)}$, where index $(n)$ corresponds to the largest $x_i$, is a complete sufficient statistic.

(b) Find a UMVU estimator for $g(\theta)$.

4. Let $(x_1, \ldots, x_n)$ be a random sample of a Poisson r.v. $X$ with unknown parameter $\lambda$.

(a) Show that

$$A_1 = \frac{1}{n} \sum_{i=1}^{n} x_i \quad \text{and} \quad A_2 = \frac{1}{2} (x_1 + x_2)$$

are both unbiased estimators of $\lambda$.

(b) Which estimator is more efficient?

5. Let $(x_1, \ldots, x_n)$ be a random sample of $X$ with mean $\mu$ and variance $\sigma^2$. A linear estimator of $\mu$ is defined to be a linear function of $x_1, \ldots, x_n$. Show that the linear estimator defined by

$$M = \frac{1}{n} \sum_{i=1}^{n} x_i = \bar{X}$$

is the most efficient linear unbiased estimator of $\mu$.

6. Let $(x_1, \ldots, x_n)$ be a random sample of a uniform r.v. $X$ over $(0, a)$, where $a$ is unknown. Show that

$$A = \max(x_1, x_2, \ldots, x_n)$$

is a consistent estimator of the parameter $a$.  

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7. Let \((x_1, \ldots, x_n)\) be a random sample of an exponential r.v. \(X\) with unknown parameter \(\lambda\). Determine the maximum-likelihood estimator of \(\lambda\). (Note: 
\(f_\lambda(x) = \lambda e^{-\lambda x}\).)

8. Let \((x_1, \ldots, x_n)\) be a random sample of a normal r.v. \(X\) with unknown mean \(\mu\) and variance 1. Assume that \(\mu\) is itself to be a normal r.v. with mean 0 and variance 1. Find the Bayes’ estimator of \(\mu\).

9. Find the mean squared estimate of a r.v. \(Y\) by a constant \(c\). (\(Y\) is an arbitrary random variable)

10. Solve these problems from the statistical inference (by Casella and Berger):

- 7.37
- 7.40
- 7.42
- 7.48
- 7.60