1. In the system shown in Figure P5.1-1, $H(e^{j\omega})$ is an ideal lowpass filter. Determine whether for some choice of input $x[n]$ and cutoff frequency $\omega_c$, the output can be the pulse

$$y[n] = \begin{cases} 
1, & 0 \leq n \leq 10, \\
0, & \text{otherwise},
\end{cases}$$

shown in Figure P5.1-2.

![Figure P5.1-1](image)

![Figure P5.1-2](image)

2. Determine the group delay for $0 < \omega < \pi$ for each of the following sequences:
   (a) $x_1[n] = \begin{cases} 
n - 1, & 1 \leq n \leq 5, \\
9 - n, & 5 < n \leq 9, \\
0, & \text{otherwise}.
\end{cases}$

   (b) $x_2[n] = \left(\frac{1}{2}\right)^{|n-1|} + \left(\frac{1}{2}\right)^{|n|}$

3. 
Consider the class of discrete-time filters whose frequency response has the form
\[ H(e^{j\omega}) = |H(e^{j\omega})|e^{-j\alpha \omega}, \]
where \(|H(e^{j\omega})|\) is a real and nonnegative function of \(\omega\) and \(\alpha\) is a real constant. As discussed in Section 5.7.1, this class of filters is referred to as linear-phase filters.

Consider also the class of discrete-time filters whose frequency response has the form
\[ H(e^{j\omega}) = A(e^{j\omega})e^{-j\alpha \omega + j\beta}, \]
where \(A(e^{j\omega})\) is a real function of \(\omega\), \(\alpha\) is a real constant, and \(\beta\) is a real constant. As discussed in Section 5.7.2, filters in this class are referred to as generalized linear-phase filters.

For each of the filters in Figure P5.15-1, determine whether it is a generalized linear-phase filter. If it is, then find \(A(e^{j\omega})\), \(\alpha\), and \(\beta\). In addition, for each filter you determine to be a generalized linear-phase filter, indicate whether it also meets the more stringent criterion for being a linear-phase filter.

![Figure P5.15-1](image_url)

Figure P5.19-1 shows the impulse responses for several different LTI systems. Find the group delay associated with each system.
Let $h_{lp}[n]$ denote the impulse response of an ideal lowpass filter with unity passband gain and cutoff frequency $\omega_c = \pi/4$. Figure P5.21-1 shows five systems, each of which is equivalent to an ideal LTI frequency-selective filter. For each system shown, sketch the equivalent frequency response, indicating explicitly the band-edge frequencies in terms of $\omega_c$. In each case, specify whether the system is a lowpass, highpass, bandpass, bandstop, or multiband filter.

(a)

(b)

(c)

(d)

(e) Figure P5.21-1
An LTI system with impulse response \( h_1[n] \) is an ideal lowpass filter with cutoff frequency \( \omega_c = \pi/2 \). The frequency response of the system is \( H_1(e^{j\omega}) \). Suppose a new LTI system with impulse response \( h_2[n] \) is obtained from \( h_1[n] \) by

\[
h_2[n] = (-1)^n h_1[n].
\]

Sketch the frequency response \( H_2(e^{j\omega}) \).

7.

Figure P5.41-1 shows two different interconnections of three systems. The impulse responses \( h_1[n], h_2[n], \) and \( h_3[n] \) are as shown in Figure P5.41-2. Determine whether system A and/or system B is a generalized linear-phase system.

![Figure P5.41-1](image)

![Figure P5.41-2](image)

8.
Let \( h_{lp}[n] \) denote the impulse response of an FIR generalized linear-phase lowpass filter. The impulse response \( h_{hp}[n] \) of an FIR generalized linear-phase highpass filter can be obtained by the transformation

\[
h_{hp}[n] = (-1)^n h_{lp}[n].
\]

If we decide to design a highpass filter using this transformation and we wish the resulting highpass filter to be symmetric, which of the four types of generalized linear-phase FIR filters can we use for the design of the lowpass filter? Your answer should consider all the possible types.

9.

The LTI systems \( H_1(e^{j\omega}) \) and \( H_2(e^{j\omega}) \) are generalized linear-phase systems. Which, if any, of the following systems also must be generalized linear-phase systems?

(a) \[
G_1(e^{j\omega}) = H_1(e^{j\omega}) + H_2(e^{j\omega})
\]

(b) \[
G_2(e^{j\omega}) = H_1(e^{j\omega})H_2(e^{j\omega})
\]

(c) \[
G_3(e^{j\omega}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_1(e^{j\theta})H_2(e^{j(\omega-\theta)})d\theta
\]

10.

The system function \( H_{II}(z) \) represents a type II FIR generalized linear-phase system with impulse response \( h_{II}[n] \). This system is cascaded with an LTI system whose system function is \( (1 - z^{-1}) \) to produce a third system with system function \( H(z) \) and impulse response \( h[n] \). Prove that the overall system is a generalized linear-phase system, and determine what type of linear phase system it is.