1. 

\[ y[n] = \begin{cases} 
1, & 0 \leq n \leq 10, \\
0, & \text{otherwise} 
\end{cases} \]

Therefore,

\[ Y(e^{j\omega}) = e^{-j\omega} \frac{\sin \frac{11\omega}{2}}{\sin \frac{\omega}{2}} \]

This \( Y(e^{j\omega}) \) is full band. Therefore, since \( Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) \), the only possible \( x[n] \) and \( \omega_c \) that could produce \( y[n] \) is \( x[n] = y[n] \) and \( \omega_c = \pi \).

2. (a) By the symmetry of \( x_1[n] \) we know it has linear phase. The symmetry is around \( n = 5 \) so the continuous phase of \( X_1(e^{j\omega}) \) is \( \text{arg}[X_1(e^{j\omega})] = -5\omega \). Thus,

\[
\text{grd}[X_1(e^{j\omega})] = -\frac{d}{d\omega} \{\text{arg}[X_1(e^{j\omega})]\} = -\frac{d}{d\omega} \{-5\omega\} = 5
\]

(b) By the symmetry of \( x_2[n] \) we know it has linear phase. The symmetry is around \( n = 1/2 \) so we know the phase of \( X_2(e^{j\omega}) \) is \( \text{arg}[X_2(e^{j\omega})] = -\omega/2 \). Thus,

\[
\text{grd}[X_2(e^{j\omega})] = -\frac{d}{d\omega} \{\text{arg}[X_2(e^{j\omega})]\} = -\frac{d}{d\omega} \{-\frac{\omega}{2}\} = \frac{1}{2}
\]

3. (a) \( h[n] \) is symmetric about \( n = 1 \).

\[
H(e^{j\omega}) = 2 + e^{-j\omega} + 2e^{-2j\omega} \\
= e^{-j\omega}(2e^{j\omega} + 1 + 2e^{-j\omega}) \\
= (1 + 4\cos\omega)e^{-j\omega}
\]
\[ A(\omega) = 1 + 4 \cos \omega, \quad \alpha = 1, \beta = 0 \]

Generalized Linear phase but not Linear Phase since \( A(\omega) \) is not always positive.

(b) This sequence has no even or odd symmetry, so it does not possess generalized linear phase.

(c) \( h[n] \) is symmetric about \( n = 1 \).

\[
H(e^{j\omega}) = 1 + 3e^{-j\omega} + e^{-2j\omega} \\
= e^{-j\omega}(e^{j\omega} + 3 + e^{-j\omega}) \\
= (3 + 2\cos \omega)e^{-j\omega}
\]

\[ A(\omega) = 3 + 2\cos \omega, \quad \alpha = 1, \beta = 0 \]

Generalized Linear phase & Linear Phase.

(d) \( h[n] \) has even symmetry.

\[
H(e^{j\omega}) = 1 + e^{-j\omega} \\
= e^{-j(1/2)\omega}(e^{j(1/2)\omega} + e^{-j(1/2)\omega}) \\
= 2\cos(\omega/2)e^{-j(1/2)\omega}
\]

\[ A(\omega) = 2\cos(\omega/2), \quad \alpha = \frac{1}{2}, \beta = 0 \]

Generalized Linear Phase but not Linear Phase since \( A(\omega) \) is not always positive.

(e) \( h[n] \) has odd symmetry.

\[
H(e^{j\omega}) = 1 - e^{-2j\omega} \\
= e^{-j\omega}(e^{j\omega} - e^{-j\omega}) \\
= e^{-j\omega}2j\sin \omega \\
= (2\sin \omega)e^{-j\omega + j\frac{\pi}{2}}
\]

\[ A(\omega) = 2\sin \omega, \quad \alpha = 1, \beta = \frac{\pi}{2} \]

Generalized Linear Phase but not Linear Phase since \( A(\omega) \) is not always positive.

4. Due to the symmetry of the impulse responses, all the systems have generalized linear phase of \( \arg[H(e^{j\omega})] = \beta - n_\omega \) where \( n_\omega \) is the point of symmetry in the impulse response graphs. The group delay is

\[
\text{grd}[H(e^{j\omega})] = -\frac{d}{d\omega} \{\arg[H(e^{j\omega})]\} = -\frac{d}{d\omega} (\beta - n_\omega) = n_\omega
\]

To find each system’s group delay we need only find the point of symmetry \( n_\omega \) in each system’s impulse response.

\[
\text{grd}[H_1(e^{j\omega})] = 2 \quad \text{grd}[H_4(e^{j\omega})] = 3 \\
\text{grd}[H_2(e^{j\omega})] = 1.5 \quad \text{grd}[H_5(e^{j\omega})] = 3 \\
\text{grd}[H_3(e^{j\omega})] = 2 \quad \text{grd}[H_6(e^{j\omega})] = 3.5
\]
5.

\( h_{lp}[n] \) is an ideal lowpass filter with \( \omega_c = \frac{\pi}{4} \)

(a) \( y[n] = x[n] - x[n] \ast h_{lp}[n] \Rightarrow H(e^{j\omega}) = 1 - H_{lp}(e^{j\omega}) \)
This is a highpass filter.

(b) \( x[n] \) is first modulated by \( \pi \), lowpass filtered, and demodulated by \( \pi \). Therefore, \( H_{lp}(e^{j\omega}) \) filters the high frequency components of \( X(e^{j\omega}) \).
This is a highpass filter.

(c) \( h_{lp}[2n] \) is a downsampling version of the filter. Therefore, the frequency response will be "spread out" by a factor of two, with a gain of \( \frac{1}{2} \).
This is a lowpass filter.
(d) This system upsamples $h_p[n]$ by a factor of two. Therefore, the frequency axis will be compressed by a factor of two.
This is a bandstop filter.

(e) This system upsamples the input before passing it through $h_p[n]$. This effectively doubles the frequency bandwidth of $H_p(e^{j\omega})$.
This is a lowpass filter.
Making use of some DTFT properties can aide in the solution of this problem. First, note that

\[
  h_2[n] = (-1)^{\nu} h_1[n]
\]

\[
  h_2[n] = e^{-j\nu n} h_1[n]
\]

Using the DTFT property that states that modulation in the time domain corresponds to a shift in the frequency domain,

\[
  H_2(e^{j\omega}) = H_1(e^{j(\omega + \nu)})
\]

Consequently, \( H_2(e^{j\omega}) \) is simply \( H_1(e^{j\omega}) \) shifted by \( \nu \). The ideal low pass filter has now become the ideal high pass filter, as shown below.

![Diagram showing the DTFT properties and frequency domain shift.]

7. Convolving two symmetric sequences yields another symmetric sequence. A symmetric sequence convolved with an antisymmetric sequence gives an antisymmetric sequence. If you convolve two antisymmetric sequences, you will get a symmetric sequence.

\[
  A : \quad h_1[n] \ast h_2[n] \ast h_3[n] = (h_1[n] \ast h_2[n]) \ast h_3[n]
\]

\( h_1[n] \ast h_2[n] \) is symmetric about \( n = 3 \), \(-1 \leq n \leq 7\)

\( (h_1[n] \ast h_2[n]) \ast h_3[n] \) is antisymmetric about \( n = 3 \), \(-3 \leq n \leq 9\)

Thus, system A has generalized linear phase

\[
  B : \quad (h_1[n] \ast h_2[n]) + h_3[n]
\]

\( h_1[n] \ast h_2[n] \) is symmetric about \( n = 3 \), as we noted above. Adding \( h_3[n] \) to this sequence will destroy all symmetry, so this does not have generalized linear phase.

8.
Filter Types II and III cannot be highpass filters since they both must have a zero at $z = 1$.

Type I $\rightarrow$ Type I could be highpass:

Type II $\rightarrow$ Type IV can be highpass:

Type III $\rightarrow$ Type III cannot be highpass:

Type IV $\rightarrow$ Type II cannot be highpass:
(a) This system does not necessarily have generalized linear phase. The phase response,

\[ G_1(e^{j\omega}) = \tan^{-1}\left( \frac{\text{Im}(H_1(e^{j\omega}) + H_2(e^{j\omega}))}{\text{Re}(H_1(e^{j\omega}) + H_2(e^{j\omega}))} \right) \]

is not necessarily linear. As a counter-example, consider the systems

\[
\begin{align*}
h_1[n] &= \delta[n] + \delta[n - 1] \\
h_2[n] &= 2\delta[n] - 2\delta[n - 1] \\
g_1[n] &= h_1[n] + h_2[n] = 3\delta[n] - \delta[n - 1] \\
G_1(e^{j\omega}) &= 3 - e^{-j\omega} = 3 - \cos \omega + j\sin \omega \\
\angle G_1(e^{j\omega}) &= \tan^{-1}\left( \frac{\sin \omega}{3 - \cos \omega} \right)
\end{align*}
\]

Clearly, \(G_1(e^{j\omega})\) does not have linear phase.

(b) This system must have generalized linear phase.

\[
\begin{align*}
G_2(e^{j\omega}) &= H_1(e^{j\omega})H_2(e^{j\omega}) \\
|G_2(e^{j\omega})| &= |H_1(e^{j\omega})| |H_2(e^{j\omega})| \\
\angle G_2(e^{j\omega}) &= \angle H_1(e^{j\omega}) + \angle H_2(e^{j\omega})
\end{align*}
\]

The sum of two linear phase responses is also a linear phase response.

(c) This system does not necessarily have linear phase. Using properties of the DTFT, the circular convolution of \(H_1(e^{j\omega})\) and \(H_2(e^{j\omega})\) is related to the product of \(h_1[n]\) and \(h_2[n]\). Consider the systems

\[
\begin{align*}
h_1[n] &= \delta[n] + \delta[n - 1] \\
h_2[n] &= \delta[n] + 2\delta[n - 1] + \delta[n - 2] \\
g_2[n] &= h_1[n]h_2[n] = \delta[n] + 2\delta[n - 1] \\
G_3(e^{j\omega}) &= 1 + 2e^{-j\omega} = 1 + 2\cos \omega - j2\sin \omega \\
\angle G_3(e^{j\omega}) &= \tan^{-1}\left( \frac{2\sin \omega}{1 + 2\cos \omega} \right)
\end{align*}
\]

Clearly, \(G_3(e^{j\omega})\) does not have linear phase.
The type II FIR system \( H_{II}(z) \) has generalized linear phase. Therefore, it can be written in the form

\[
H_{II}(e^{j\omega}) = A_\omega(e^{j\omega})e^{-j\omega M/2}
\]

where \( M \) is an odd integer and \( A_\omega(e^{j\omega}) \) is a real, even, periodic function of \( \omega \). Note that the system \( G(z) = (1 - z^{-1}) \) is a type IV generalized linear phase system.

\[
G(e^{j\omega}) = 1 - e^{-j\omega} = e^{-j\omega/2}(e^{j\omega/2} - e^{-j\omega/2}) = e^{-j\omega/2}(2j\sin(\omega/2)) = 2\sin(\omega/2)e^{-j\omega/2 + j\pi/2} = A_\omega(e^{j\omega})e^{-j\omega/2 + j\pi/2}
\]

\[
A_\omega(e^{j\omega}) = 2\sin(\omega/2)
\]

\[
\angle G(e^{j\omega}) = -\frac{\omega}{2} + \frac{\pi}{2}
\]

The cascade of \( H_{II}(z) \) with \( G(z) \) results in a generalized linear phase system \( H(z) \).

\[
H(e^{j\omega}) = A_\omega(e^{j\omega})A_\omega(e^{j\omega})e^{-j\omega M/2}e^{-j\omega/2 + j\pi/2} = A'_\omega(e^{j\omega})e^{j\omega M'/2 + j\pi/2}
\]

where \( A'_\omega(e^{j\omega}) \) is a real, odd, periodic function of \( \omega \) and \( M' \) is an even integer.

Thus, the resulting system \( H(e^{j\omega}) \) has the form of a type III FIR generalized linear phase system. It is antisymmetric, has odd length (\( M \) is even), and has generalized linear phase.