Problem 1:

In this problem, we wish to design a FIR length-7 linear phase multi-band type-1 filter for the spectrum shown below:

\[
|H_d(e^{j\omega})| \quad \omega = \frac{\pi}{4}, \frac{\pi}{2}, \frac{3\pi}{4}, \pi
\]

Design the 7-tap FIR filter using two approaches: first using frequency sampling approach, and then by using a Fourier series approach using rectangular window. Let \(h_1[n]\) represent the 7-tap causal frequency sampling filter with spectrum \(H_1(e^{j\omega})\) and \(h_2[n]\) represent the 7-tap causal rectangular window filter with spectrum \(H_2(e^{j\omega})\). Calculate \(h_1[n]\) and \(h_2[n]\). Which of these two filters has the minimum least squared integral error? Let

\[
E_1 = \int_{-\pi}^{\pi} |H_1(e^{j\omega}) - e^{-j\omega}H_d(e^{j\omega})|^2 \, d\omega,
\]

and

\[
E_2 = \int_{-\pi}^{\pi} |H_2(e^{j\omega}) - e^{-j\omega}H_d(e^{j\omega})|^2 \, d\omega.
\]

In the above least square integral error expressions, \(H_d(e^{j\omega})\) is the zero-phase desired filter. Calculate \(E_1, E_2, \) and \(E_1 - E_2\).

Problem 2:

Consider the design of the multi-band filter spectrum in Problem 1 using a causal length-7 linear-phase real FIR filter using frequency sampling with 11-point oversampling. Let this filter be \(h_3[n]\). Calculate \(E_1 - E_2\) where \(E_4\) is defined in Problem 1 and

\[
E_3 = \int_{-\pi}^{\pi} |H_3(e^{j\omega}) - e^{-j\omega}H_d(e^{j\omega})|^2 \, d\omega.
\]

Problem 3:

Design a length-3 FIR filter which meets (or exceeds) the following specifications:

(a) Linear phase (constant group delay of 1 sample)
(b) The magnitude, \(|H(e^{j2\pi F})|\) should be

(i) smaller than 0.3 for \(0.25 \leq F \leq 0.5\)
(ii) within 0.3 of 1.0 for \(0 \leq F \leq 0.125\); i.e., \(|1 - |H(e^{j2\pi F})|| < 0.3\) for \(0 \leq F \leq 0.125\).

Sketch the magnitude \(|H(e^{j2\pi F})|\) and impulse response \(h[n]\).
Problem 4:

The original Parks-McClellan algorithm worked only for Type I filter designs. Other filter types can be designed by converting them to Type I design problems. Consider the system function

$$\tilde{H}(z) = (1 + z^{-1}) H_1(z)$$

where $H_1(z)$ is a Type I filter. The design problem can then be stated as the problem of finding the set of filter coefficients that minimize the error function

$$E(\omega) = W(\omega) \left[ H_d(e^{j\omega}) - (1 + e^{-j\omega}) H_1(e^{j\omega}) \right]$$

$$= W(\omega) (1 + e^{-j\omega}) \left[ H_d(e^{j\omega})/(1 + e^{-j\omega}) - H_1(e^{j\omega}) \right]$$

which is in the form of a Type I equiripple design problem with a different ideal function and a different frequency weighting.

(a) What type of filter is $\tilde{H}(e^{j\omega})$?

(b) Let $h_1[n]$ denote the solution to the above Type I approximation problem with the modified error function. Determine a formula for $\hat{h}[n]$, the solution to the other filter type problem, in terms of $h_1[n]$.

(c) If $\hat{h}[n]$ is a $2N$-point FIR lowpass filter, what is the minimum number of alternations that it can possess?

Problem 5:

Work problem 7.15 from the text book.

Problem 6:

Work problem 7.16 from the text book.

Problem 7:

Work problem 7.30 from the text book.

Problem 8:

Work problem 7.36 from the text book.

Good Luck