1.

The bilinear transformation is used to design an ideal discrete-time lowpass filter with cutoff frequency \( \omega_c = 3\pi / 5 \) from an ideal continuous-time lowpass filter with cutoff frequency \( \Omega_c = 2\pi \cdot 300 \) rad/s. Give a choice for the parameter \( T \) that is consistent with this information. Is this choice unique? If not, give another choice which is consistent with the information.

2.

A discrete-time lowpass filter is to be designed by applying the impulse invariance method to a continuous-time Butterworth filter having magnitude-squared function

\[
|H_c(j\Omega)|^2 = \frac{1}{1 + (\Omega / \Omega_c)^{2N}}.
\]

The specifications for the discrete-time system are those of Example 7.2, i.e.,

\[
0.89125 \leq |H(e^{j\omega})| \leq 1, \quad 0 \leq |\omega| \leq 0.2\pi,
\]

\[
|H(e^{j\omega})| \leq 0.17783, \quad 0.3\pi \leq |\omega| \leq \pi.
\]

Assume, as in that example, that aliasing will not be a problem; i.e., design the continuous-time Butterworth filter to meet passband and stopband specifications as determined by the desired discrete-time filter.

(a) Sketch the tolerance bounds on the magnitude of the frequency response, \( |H_c(j\Omega)| \), of the continuous-time Butterworth filter such that after application of the impulse invariance method (i.e., \( h[n] = T_d h_c(nT_d) \)), the resulting discrete-time filter will satisfy the given design specifications. Do not assume that \( T_d = 1 \) as in Example 7.2.

(b) Determine the integer order \( N \) and the quantity \( T_d \Omega_c \) such that the continuous-time Butterworth filter exactly meets the specifications determined in Part (a) at the passband edge.

(c) Note that if \( T_d = 1 \), your answer in Part (b) should give the values of \( N \) and \( \Omega_c \) obtained in Example 7.2. Use this observation to determine the system function \( H_c(s) \) for \( T_d \neq 1 \) and to argue that the system function \( H(z) \) which results from impulse invariance design with \( T_d \neq 1 \) is the same as the result for \( T_d = 1 \) given by Eq. (7.19).
3. We wish to use impulse invariance or the bilinear transformation to design a discrete-time filter that meets specifications of the following form:

\[ 1 - \delta_1 \leq |H(e^{j\omega})| \leq 1 + \delta_1, \quad 0 \leq |\omega| \leq \omega_p, \]
\[ |H(e^{j\omega})| \leq \delta_2, \quad \omega_s \leq |\omega| \leq \pi. \]  

(P7.3-1)

For historical reasons, most of the design formulas, tables, or charts for continuous-time filters are normally specified with a peak gain of unity in the passband; i.e.,

\[ 1 - \delta_1 \leq |H_c(j\Omega)| \leq 1, \quad 0 \leq |\Omega| \leq \Omega_p, \]
\[ |H_c(\Omega)| \leq \delta_2, \quad \Omega_s \leq |\Omega|. \]  

(P7.3-2)

Useful design charts for continuous-time filters specified in this form were given by Rabiner, Kaiser, Herrmann, and Dolan (1974).

(a) To use such tables and charts to design discrete-time systems with a peak gain of \((1 + \delta_1)\), it is necessary to convert the discrete-time specifications into specifications of the form of Eq. (P7.3-2). This can be done by dividing the discrete-time specifications by \((1 + \delta_1)\). Use this approach to obtain an expression for \(\delta_1\) and \(\delta_2\) in terms of \(\delta_1\) and \(\delta_2\).

(b) In Example 7.2, we designed a discrete-time filter with a maximum passband gain of unity. This filter can be converted to a filter satisfying a set of specifications such as those in Eq. (P7.3-1) by multiplying by a constant of the form \((1 + \delta_1)\). Find the required value of \(\delta_1\) and the corresponding value of \(\delta_2\) for this example, and use Eq. (7.19) to determine the coefficients of the system function of the new filter.

(c) Repeat Part (b) for the filter in Example 7.3.

4. Suppose that we wish to design a highpass filter satisfying the following specification:

\[ -0.04 < |H(e^{j\omega})| < 0.04, \quad 0 \leq |\omega| \leq 0.2\pi, \]
\[ 0.995 < |H(e^{j\omega})| < 1.005, \quad 0.3\pi \leq |\omega| \leq \pi. \]

The filter will be designed using the bilinear transformation and \(T = 2\) ms with a prototype continuous-time filter. State the specifications that should be used to design the prototype continuous-time filter to ensure that the specifications for the discrete-time filter are met.
5. Problem 7.28 from textbook.

6. Problem 7.29 from textbook.

7. Problem 7.30 from textbook.

8. Problem 7.31 from textbook.