Homework 4

Problems

1. Each of the members of a 7-judge panel independently makes a correct decision with probability 0.7. If the panel’s decision is made by majority rule, what is the probability that the panel makes the correct decision? Given that 4 of the judges agreed, what is the probability that the panel made the correct decision?

2. A fair coin is tossed 900 times. Find the probability that the number of heads is between 420 and 465.

3. If you buy a lottery ticket in 50 lotteries, in each of which your chance of winning a prize is $1/100$, what is the (approximate) probability that you will win a prize (a) at least once, (b) exactly once, (c) at least twice?

4. A 300-page look contains 200 typos. Calculate, using Poisson distribution, the probability that a particular page contains at least two typos.

5. Let the probability density of $X$ be given by

$$f(x) = \begin{cases} c(4x - 2x^2), & 9 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$

(a) What is the value of $c$?
(b) $P\left\{\frac{1}{2} < X < \frac{3}{2}\right\}$?

6. A total of $r$ keys are to be put, one at a time, in $k$ boxes, which each key independently being put in box $i$ with probability $p_i$, $\sum_{i=1}^{k} p_i = 1$. Each time a key is put in a nonempty box, we say that collision occurs. Find the expected number of collisions.

7. If $X$ is a nonnegative integer valued random variable, show that

$$E[X] = \sum_{n=1}^{\infty} P\{X \geq n\}$$

8. Let $X_1$ and $X_2$ be independent geometric random variables having the same parameter $p$. Guess the value of

$$P\{X_1 = i \mid X_1 + X_2 = n\}$$

Hint: Suppose a coin having probability $p$ of coming up heads is continually flipped. If the second head occurs on flip number $n$, what is the conditional probability that the first head was on flip number $i$, $i = 1, ..., n - 1$?

Verify yours guess analytically.